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Optimal unemployment insurance: Consumption versus expenditure☆

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HIGHLIGHTS

- We study optimal unemployment insurance if the unemployed pay lower prices.
- We derive a sufficient statistics formula in terms of observable variables.
- We compare our results to the standard Baily–Chetty formula.
- Lower insurance is optimal if relative risk aversion is greater than one.
- We calculate optimal replacement ratios for the United States.

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ABSTRACT

We study the optimal provision of unemployment insurance (UI) in a framework that distinguishes between consumption and expenditure. We derive a "sufficient statistics" formula for optimal UI that is expressed in terms of observable variables and can therefore be used in applied work. Recent research has shown that unemployed households pay less per unit of consumption than employed households. This finding has two counteracting effects on the optimal level of UI. On the one hand, consumption smoothing benefits identified from expenditure data overestimate the true marginal benefits of UI. On the other hand, UI benefits become more valuable because they buy more consumption when unemployed. In an optimal design, which effect dominates depends on the curvature of the utility function. We show that for relative risk aversion larger than one the first effect dominates, leading to lower levels of optimal UI.

1. Introduction

A central question in an unemployment insurance scheme is how generous the program should be. Because providing consumption insurance distorts incentives to search for a new job, optimal insurance design takes into account the efficiency costs induced by moral hazard. These costs must be balanced against the welfare gains brought about by insuring workers against consumption drops due to unemployment. From an empirical perspective, the potential welfare gains of consumption-smoothing can be quantified by the drop in consumption experienced upon unemployment. The size of this drop has been estimated time and again—sometimes without a direct reference to unemployment insurance—for example by Cochrane (1991), Gruber (1997), Browning and Crossley (2001), and Stephens (2001). A characteristic shared by these studies is that, because of the data available, they focus on consumption expenditure (price times quantity) rather than on consumption (quantity).

There is now a host of evidence that indicates that the unemployed pay lower prices than their employed counterparts. This evidence suggests that activities such as shopping and searching for bargains play a role in lowering prices. Using time use surveys, Aguiar et al. (2013) find that the unemployed devote more time to shopping: in the United States, roughly 7% of the time freed up from work is dedicated to activities such as shopping for groceries and other household items, comparison shopping, coupon clipping, and buying goods online. In comparison, only between 2 and 6% of the time freed up is used to increase job search. Krueger and Mueller (2012) corroborate this finding in an international sample by studying time use surveys from 14 different countries.

Increased shopping time translates into lower prices. Using supermarket scanner data, Aguiar and Hurst (2007) verify that increased shopping effort lowers the price paid for grocery items

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while maintaining quality constant in the general population. Aguiar and Hurst (2005) focus specifically on the unemployed and find that expenditure by the unemployed falls more than consumption, indicating a reduction in the price paid per unit of consumption.

In this paper we study how distinguishing between expenditure and consumption affects the level of optimal unemployment insurance. Acknowledging that expenditure is not the same as consumption has two countervailing effects on the optimal benefit level. On the one hand, optimal unemployment insurance takes into account that the unemployed have access to lower prices in the unemployed state. Because a given dollar amount buys more consumption in the unemployed state, from the perspective of a benevolent social planner, it becomes worthwhile to transfer income from the good to the bad state. This effect tilts the balance in favor of more generous unemployment benefits. On the other hand, estimations that rely on expenditure data will overestimate the consumption-smoothing benefit of unemployment insurance because they disregard the change in prices. A correct measurement therefore tilts the balance in the direction of lower optimal unemployment benefits.

We formalize these ideas by adapting the standard normative model of social insurance originally due to Baily (1978). Chetty (2006) showed that Baily's setup captures the main trade-offs that arise in fully intertemporal settings in the style of those considered by Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). The Baily–Chetty model is part of a general class of models in public economics in which optimal policies can be computed from a reduced number of sufficient statistics.[1] In this model, optimal unemployment benefits are described by a simple formula that involves only three sufficient statistics: the magnitude of the consumption drop experienced at unemployment, the level of relative risk-aversion, and the elasticity of unemployment duration with respect to the benefit level.

We depart from the Baily–Chetty model by allowing agents to endogenously choose how much of the time freed up by not working they devote to shopping activities. By increasing shopping time they reduce prices paid for consumption. The social planner sets optimal unemployment benefits taking into account responses by workers, in particular the endogenous choice of shopping time. As usual in this literature, because of the Envelope Theorem, several endogenous choices do not have a first-order effect on optimal benefit levels. An important feature of our model is that shopping affects state-prices and therefore the implicit return of transferring resources across states faced by the social planner.

In comparison to the standard first order condition of the social planner in the Baily–Chetty model, marginal utility of the worker in the unemployed state is scaled upward by the gross return of transferring resources across states. This first order condition could be used to inform policy on the optimal level of unemployment insurance if consumption was directly observable. Because consumption is usually not observed in real-world data, we show how the optimality condition can be re-expressed in terms of expenditure.

If the worker's preferences can be described by a constant relative risk-aversion (CRRA) utility function, then optimal policy can be expressed in terms of expenditure in a generalized version of the standard Baily–Chetty formula. Whether optimal benefit levels obtained from expenditure data should be revised upward or downward depends exclusively on the degree of relative risk aversion. We show that, under the usual assumption of relative risk aversion greater than one, estimations based on expenditure data systematically overestimate the level of optimal unemployment benefits.

To illustrate the empirical relevance of our theoretical result, we calibrate our formula to US data following the approach of Gruber (1997). We find that, when compared to the standard Baily–Chetty formula, if the price paid by the unemployed is 5% lower than what the employed pay for their consumption, then, for levels of risk aversion slightly above two, replacement rates are at least 10 percentage points lower. For example, for a level of relative risk-aversion of 3, the optimal replacement rate according to the standard formula is 38.6% whereas it is 25.7% in the formula that takes into account the distinction between consumption and expenditure.

Because of the way the distinction between consumption and expenditure enters the social planner's maximization problem, it will be of relevance not only for the canonical Baily–Chetty model but also for the class of social insurance models that generalize their environment and obtain similar "sufficient statistics" formulas. We show how the wedge introduced by the distinction between consumption and expenditure can inform these related models. Notably, this wedge does not depend on the elasticity measuring the behavior that leads to moral hazard by the insured population. It is, however, influenced by the difference in prices paid in the good and the bad state of the world. Therefore, empirical research on economic shocks that distinguishes between consumption and expenditure holds important insights for social insurance in general.

The remainder of the paper is organized as follows. In the next section we develop the model, derive a "sufficient statistics" formula for optimal unemployment insurance, and show how this formula can be expressed in terms of observable variables with CRRA preferences. In Section 3 we apply the model to calculate the optimal replacement ratio for the US. In Section 4 we explore whether some of our assumptions can be relaxed and their likely effect on our results. We conclude in Section 5. All proofs are given in the Appendix.

2. Model

We build on the two-period model used by Baily (1978) and Chetty (2006) to derive their formula for optimal unemployment insurance. We extend the Baily–Chetty model by distinguishing between consumption and expenditure and by allowing for additional uses of free time. Unemployed workers can choose to use their free time on activities that lower the price they pay for consumption.

2.1. The environment

There are two dates: 0 and 1. A risk-averse worker, who derives utility from consumption and leisure, arrives at date 0 with assets A and lives for one period until date 1. At date 0, the worker may become unemployed with exogenous probability π and stays employed with probability $1-\pi$. If employed, the worker supplies one unit of labor and obtains a wage rate w. A worker who becomes unemployed stays unemployed for a fraction of time $D \in [0, 1]$, the unemployment duration, during which labor earnings are zero.

As in the Baily–Chetty model, unemployment insurance is parameterized by the pair (b, τ), where b denotes the benefit received by an unemployed agent and τ is the tax paid (only by fully employed workers) to sustain the insurance scheme. To maintain a balanced budget, the unemployment insurance scheme must satisfy

$$(1-\pi)\tau = \pi bD. \tag{1}$$

A benevolent planner chooses the parameters of the social insurance scheme so as to maximize the worker's welfare while maintaining budget balance and taking into account the worker's optimal response to the social insurance parameters b and τ.

2.2. Time allocation

Unemployed workers can influence the unemployment duration D by varying their job search effort. Chetty (2006) did not directly

[1] A detailed analysis of the advantages and disadvantages of the "sufficient statistics" approach, as well as the history of its use in public economics, is provided by Chetty (2009). The famous dead-weight loss calculation by Harberger (1964) is an early example of this approach.

model time use when unemployed and posited a function that indirectly embeds the time costs of search and utility of leisure. In his model, this function is assumed to be increasing in D and concave. Because we consider the case in which agents have an additional use for their time, we focus in more detail on the allocation of time when unemployed.

During the duration of unemployment D, the agent can allocate the time freed up by not working to three alternative activities: leisure ℓ, job search t_D, and price search t_p. These variables are measured as additional time spent on these activities net of time already spent on them in the employed state. The time spent on these three activities is limited by the following time constraint:

$$\ell + t_D + t_p \leq D. \tag{2}$$

That is, the sum of additional time spent on all three activities is bounded by total time available D, the duration of unemployment.[2]

Leisure has a direct payoff in utility terms. Utility derived from the extra leisure when unemployed is captured by the function $\nu(\ell)$, with $\nu'(\cdot) > 0$ and $\nu''(\cdot) \leq 0$. We normalize $\nu(0) = 0$, so that $\nu(\ell)$ measures utility gains over leisure enjoyed in the employed state.

Time spent on job search reduces the duration of unemployment. As in the model of Chetty (2006), we assume that the agent controls the duration D deterministically. This is captured by the relationship $D = \delta(t_D)$. This function $\delta : [0, 1] \rightarrow [0, 1]$ satisfies $\delta'(\cdot) < 0$ and $\delta''(\cdot) > 0$. Time spent on job search reduces the duration at a decreasing rate. Analogously, time spent on the search for lower prices t_p converts into prices according to a function $p = \sigma(t_p)$, with $\sigma'(\cdot) < 0$ and $\sigma''(\cdot) > 0$. Shopping time reduces prices, albeit at a decreasing rate. This functional form is directly motivated by the existing literature that relates shopping effort to purchase prices (e.g., Aguiar and Hurst, 2007).[3]

We normalize the price of the consumption good in the employed state to 1, so that $p \leq 1$ represents the relative price per unit of consumption in the unemployed state.[4] Moreover, even if the agent uses all the available time on price search, goods cannot be obtained for free. This is formalized by introducing a lower bound for prices at $\underline{p}>0$, so that the function $\sigma : [0,1] \rightarrow [\underline{p}, 1]$. This lower bound on prices is important in ensuring that consumption in the unemployed state is bounded away from infinity in the agent's maximization problem.

2.3. Sellers of consumption goods

We do not explicitly model the sellers of the goods purchased by workers. We assume that free entry and exit produces a zero-profit condition for these sellers, so that seller's profits (and welfare) are not affected by the choices of the worker or the planner. In Section 4 we discuss how relaxing the zero-profit condition affects the results of the model.

2.4. The worker's and the planner's problem

Let $u(c)$ denote utility over consumption c, where u is a strictly concave and state-independent function. Taking the unemployment insurance parameter pair (b, τ) as given, the worker chooses consumption in the employed state c_e, consumption in the unemployed state c_u, and time allocations when unemployed $\{\ell, t_D, t_p\}$ to maximize expected utility defined as

$$(1-\pi)u(c_e) + \pi\{u(c_u) + \nu(\ell)\}. \tag{3}$$

The choices of the worker are constrained by the time constraint (2) and two budget constraints, one for each state,

$$A + (w-\tau) - c_e \geq 0, \tag{4}$$

$$A + bD + w(1-D) - pc_u \geq 0. \tag{5}$$

The worker also takes into account how t_D and t_p affect D and p through the functions

$$D = \delta(t_D), \quad p = \sigma(t_p). \tag{6}$$

In the budget constraint for the unemployed state (5) we have implicitly assumed that consumption can be obtained at price p during the whole period, and not only for the duration of unemployment.[5]

The planner maximizes indirect utility of workers defined over the unemployment insurance parameter pair (b, τ) subject to the balanced budget constraint (1).

2.5. A simplified worker's problem

To make the problem formally comparable to the model of Chetty (2006), we show how the worker's problem can be simplified. Nonsatiation in leisure implies that Eq. (2) will hold with equality. Moreover, because the functions δ and σ are strictly decreasing they admit an inverse function. Using the time constraint, leisure can be expressed in terms of these inverse functions and the variables D and p:

$$\ell(D, p) = D - \delta^{-1}(D) - \sigma^{-1}(p). \tag{7}$$

Using Eq. (7), the leisure term in utility when unemployed can be expressed as a function of D and p:

$$\nu(\ell(D, p)) \equiv \psi(D, p). \tag{8}$$

The additive separability in Eq. (7), together with functional forms of ν, δ, and σ, imply for $\psi(D, p)$ that $\psi_D > 0$, $\psi_{DD} < 0$, $\psi_p > 0$, $\psi_{pp} < 0$, and $\psi_{Dp} \leq 0$.[6]

Using this substitution, the worker's problem can be restated as the choice of consumption when employed c_e, consumption when unemployed c_u, the duration of unemployment $D \in [0, 1]$, and the price $p \in [\underline{p}, 1]$ to maximize

$$(1-\pi)u(c_e) + \pi\{u(c_u) + \psi(D, p)\} \tag{9}$$

subject to

$$A + (w-\tau) - c_e \geq 0 \tag{10}$$

$$A + bD + w(1-D) - pc_u \geq 0. \tag{11}$$

Formally, the worker's problem differs from the model of Chetty (2006) in the additional choice variable p that appears in two places in the worker's problem: in the utility function inside the term $\psi(D, p)$ and in the budget constraint of the unemployed. The model of Chetty (2006) is obtained as the special case in which $p = 1$ (meaning $t_p = 0$). In this case, $\psi(D, 1)$ varies only with D (and is strictly increasing

2 For employed workers this constraint holds trivially with $D = \ell = t_D = t_p = 0$.

3 Aguiar and Hurst (2007) postulate (and empirically verify in their data) a specification in which the price paid is decreasing in shopping time, with the returns to shopping diminishing as shopping intensity increases.

4 Because for employed agents $t_p = 0$, in terms of the function σ, this implies the normalization $\sigma(0) = 1$.

5 Although we do not pursue this issue, the alternative assumption, that cheaper prices are only available during the duration of unemployment, can be accommodated by modifying the function σ, and letting it depend on D.

6 Whether this last inequality is strict depends on the concavity of the leisure term in the utility function. If $v''(\cdot) < 0$, then $\psi_{Dp} < 0$; if $v''(\cdot) = 0$, then $\psi_{Dp} = 0$. In this last case, v is linear and $\psi(D, p)$ can be written as the sum of two separate increasing and strictly concave functions $\psi_1(D)$ and $\psi_2(p)$.

and concave in this variable), and the relative price p effectively disappears from the budget constraint.

2.6. A characterization of optimal unemployment insurance

Let $V(b, \tau)$ stand for the indirect utility of the worker. The optimal unemployment insurance scheme that satisfies budget balance is for the planner to choose b and τ to maximize $V(b, \tau)$ subject to the balanced budget constraint in Eq. (1). The following proposition characterizes the first order condition of the social planner's problem.

Proposition 1. *The marginal net benefit of a balanced-budget increase of b is given by*

$$\frac{dV(b,\tau(b))}{db} = \pi D\left[\frac{1}{p}u'(c_u) - (1+\varepsilon_{D,b})u'(c_e)\right], \tag{12}$$

where $\varepsilon_{D,b}$ is the elasticity of the duration of unemployment with respect to the unemployment insurance benefit b:

$$\varepsilon_{D,b} = \frac{b}{D}\frac{dD}{db}. \tag{13}$$

Proof: in Appendix C.

The resulting equation is formally equivalent to a formula with a state-dependent utility function in which the marginal utility in the unemployed state is (endogenously) shifted upward. Because purchasing consumption in the unemployed state costs only $p \le 1$, transferring resources from the employed to the unemployed state has an endogenous gross return of $\frac{1}{p} \ge 1$. Unless unemployed workers decide to spend absolutely no time on shopping, this gross return is strictly greater than one.

In terms of economic intuition, the expression in Eq. (12) states that the value of an additional dollar in the unemployed state is proportional to the gross return of transferring resources into that state times the marginal utility of consuming in that state: $\frac{1}{p}u'(c_u)$. The cost of providing this additional dollar is the consumption foregone in the employed state, which is measured by $u'(c_e)$ plus the behavioral effect $u'(c_e)\varepsilon_{D,b}$ that takes into account reduced job search effort.

In the case of an interior optimum, b^* is implicitly defined by setting $\frac{dV(b)}{db} = 0$. This yields the following optimality condition:

$$\frac{u'(c_u)}{u'(c_e)} = p\left[1+\varepsilon_{D,b}\right]. \tag{14}$$

At the optimal benefit level the marginal rate of substitution is equal to the behavioral elasticity of Baily (1978) and Chetty (2006) times the ratio of the endogenously chosen state-prices p.

As we foreshadowed in the introduction, when consumption is distinguished from expenditure, there are two opposing forces that could push optimal unemployment insurance either higher or lower. In our characterization so far only the first of these forces is present. The introduction of $\frac{1}{p}$ in Eq. (12) amplifies the benefit of consumption in the unemployment state, generating higher optimal unemployment benefits relative to the Baily–Chetty model (in which $p = 1$). This result is also apparent from Eq. (14); the gap between c_u and c_e induced by the behavioral elasticity is counteracted by p. When p is less than one, the marginal rate of substitution on the left hand side of Eq. (14) is lower than in the Baily–Chetty model, indicating an optimal substitution of consumption from the employed to the unemployed state, and therefore higher unemployment insurance.

Notably absent from the expression in Eq. (14) is the welfare cost related to reducing leisure in order to free up time for shopping. This is not because it was not taken into account. A first order condition for the unemployed worker equates the disutility from spending more time on price search to the benefits arising from relaxing the budget constraint in the unemployed state of the world.[7] The reason is that the variables showing up in Eq. (14) are sufficient statistics for the optimal benefit level. As is usual in the "sufficient statistics" literature, all the information necessary to solve for the optimal benefit level b^* is embedded in the equilibrium values of a relatively small number of variables.

2.7. A characterization of optimal unemployment insurance in terms of consumption expenditure: the CRRA case

The optimality condition in Eq. (14) is not expressed in terms of observational data. Because the empirical relationship between consumption and unemployment is usually estimated from data on consumption expenditure, not consumption, we re-express the equation in terms of observables. To do so, in what follows, we assume that preferences can be described by a CRRA utility function with constant relative risk aversion $\gamma > 0$.

Assumption 1. (CRRA) $u'(c) = c^{-\gamma}, \gamma > 0$.

Expenditure in the unemployed state is $\tilde{c}_u = pc_u$ and expenditure in the employed state is $\tilde{c}_e = c_e$. With the CRRA specification, the optimality condition in Eq. (14) can be expressed purely in terms of observable expenditure data.

Proposition 2. *With a CRRA specification (*Assumption 1*), at an interior optimum, the level of optimal unemployment benefits is implicitly defined by the following relationship:*

$$\gamma\Delta\log\tilde{c} = \log(1+\varepsilon_{D,b}) + (1-\gamma)\log p, \tag{15}$$

where $\Delta\log\tilde{c} \equiv \log\tilde{c}_e - \log\tilde{c}_u$ is the log-difference of expenditure between the employed and the unemployed state.

A first order approximation to this condition is

$$\gamma\frac{\Delta\tilde{c}}{\tilde{c}} \approx \varepsilon_{D,b} + (\gamma-1)(1-p), \tag{16}$$

where $\Delta\tilde{c} = \tilde{c}_e - \tilde{c}_u$ is the difference of expenditure between the employed and the unemployed state.

Proof: in Appendix C.

The expression in Eq. (15) is an exact relationship whereas Eq. (16) is an approximation. Our formula generalizes the standard Baily–Chetty formula. Notice that if either $p = 1$ or $\gamma = 1$ (in either the exact or approximate expression), then the price p does not play a role and the Baily–Chetty formula is obtained as a special case of Eq. (16).

The case in which $p = 1$ has a straightforward explanation; if the unemployed are unwilling or unable to lower the prices paid, because they do not spend time searching for lower prices, then the distinction between consumption and expenditure becomes irrelevant for the optimal choice of b. The second case, in which $\gamma = 1$, has a less mechanical explanation. When the degree of risk-aversion is 1, preferences are equivalent to a Cobb–Douglas utility function, which has a unitary elasticity of substitution. Therefore, optimal expenditure choices do not respond to changes in relative prices and, again, the distinction between consumption and expenditure becomes irrelevant for the optimal choice of b.

If $\gamma \neq 1$ then optimal expenditure choices do respond to changes in relative prices. The social planner internalizes the preferences of workers and therefore inherits their desire to smooth or substitute consumption across states. At low levels of relative risk aversion, when $\gamma < 1$, workers are relatively uninterested in smoothing consumption and are willing to substitute from the good to the bad state of the

[7] Assuming interior solutions, in the worker's problem the first order condition with respect to p implies $\psi_p = \lambda_u c_u$. Due to the Envelope Condition, the effect involving leisure drops out of the determination of the optimal benefit because $\frac{dV(b)}{dp} = \psi_p = \lambda_u c_u$ is zero at the optimum.

world if the gross return of doing so, $\frac{1}{p}$, is greater than one. Therefore, with $\gamma < 1$ the social planner will choose optimal benefit levels b^* that exceed those obtained from the standard formulation in the Baily–Chetty model. This is apparent from either Eq. (15) or Eq. (16), where, with $\gamma < 1$, the last term on the right hand side of either equation is negative. Relative to the standard Baily–Chetty formula, which omits this last term, the planner optimally chooses a consumption expenditure gap that is lower and therefore a higher degree of insurance. However, virtually all calibrations consider levels of relative risk-aversion $\gamma > 1$ more realistic. In this case, the result is reversed, and our formula deviates from the standard formulation in the direction of lower optimal benefit levels b^*, and estimations based on expenditure data systematically over-estimate the level of optimal unemployment benefits. We summarize this discussion in the following corollary.

Corollary 1. *Whenever $p < 1$, for preferences described by a CRRA utility function with risk aversion parameter γ, the sign of $\gamma - 1$ fully determines whether directly applying the Baily–Chetty formula to expenditure data overstates or understates the level of optimal unemployment insurance. If $\gamma < 1$, then the standard Baily–Chetty formula understates the level of optimal unemployment insurance. If $\gamma > 1$, then the standard Baily–Chetty formula overstates the level of optimal unemployment insurance.*

Summing up, as we mentioned in our introduction, there are two opposing forces that could push optimal unemployment insurance either higher or lower. The first force pushed optimal unemployment insurance higher because a dollar buys more consumption in the unemployed state. In this section we have focused on the second force, which pushes optimal unemployment insurance lower because expenditure drops overestimate consumption drops when part of the movement in expenditures is due to prices. Therefore, consumption-smoothing benefits measured from expenditure are overestimated. To consider both forces simultaneously, we expressed the optimality condition in Eq. (14) in terms of expenditure rather than consumption. Which of the two forces prevails depends on how strong the worker's desire to smooth consumption across the employed and the unemployed state is. With a CRRA specification this question can be answered precisely by focusing on a single parameter: the degree of relative risk aversion.

3. Application: the optimal replacement ratio

To gauge the practical importance of the distinction between consumption and expenditure we apply our condition describing the optimal unemployment insurance benefit to the United States. We use the approach by Gruber (1997), who uses consumption expenditure data to arrive at optimal replacement rates. Gruber's method has been extremely influential and has been widely used in applied work bearing on unemployment insurance.[8]

Gruber (1997) estimates a linear relationship between the expenditure drop at unemployment and the replacement rate $r = \frac{b}{w}$ of the form

$$\Delta \log \tilde{c} = \alpha + \beta r. \tag{17}$$

In this expression, $\alpha > 0$ measures the gap of expenditure across employment states in the absence of unemployment benefits, and $\beta < 0$ measures how this gap is narrowed as the replacement rate increases. Once these parameters have been estimated, substituting Gruber's specification into the optimality condition in Eq. (15) permits to isolate the optimal replacement ratio r^* as a function of the relative risk aversion parameter γ, the elasticity of the duration with respect to benefits $\varepsilon_{D,b}$, and the relative price p of a unit of consumption in the unemployed state:

$$r^*(\gamma, p) = \frac{1}{\hat{\beta}}\left[-\hat{\alpha} + \frac{1}{\gamma}\log(1+\varepsilon_{D,b}) + \frac{1-\gamma}{\gamma}\log p\right]. \tag{18}$$

In our calculations we adopt the baseline estimates by Gruber (1997): $\hat{\alpha} = 0.222$ and $\hat{\beta} = -0.265$. Gruber (1997) reports an average replacement rate of 42.6%. With these estimates, consumption expenditure at average replacement rates falls by roughly 10% when a worker enters unemployment.[9] We follow the literature and set the duration elasticity $\varepsilon_{D,b} = 0.500$. This value is based on the survey by Krueger and Meyer (2002).[10] We present results for a range of levels of relative risk-aversion γ that go from 1 to 5.

To calibrate p, we use the results by Aguiar and Hurst (2005), who estimate the effect of unemployment on food expenditure and food consumption for a cross-section of US households. Based on our reading of their findings, our preferred estimate for p is 0.950, i.e., a 5% drop in the prices paid by the unemployed, although our choice merits some discussion.

Aguiar and Hurst (2005) construct a consumption index to map quantity data into a permanent income measure. In their sample, unemployment reduces food expenditure by 19% and food consumption by 5%. This implies that in their data almost three-quarters of the drop in expenditure are due to a lower price per unit of consumption. With an average drop of 10% in expenditure caused by unemployment, the corresponding value for p would be 0.925. However, the 19% drop in expenditure due to unemployment estimated by Aguiar and Hurst (2005) is relatively large compared to the usual estimates, which hover around 10% (e.g., Stephens, 2001). Assuming that their consumption index is correctly estimated, we therefore believe that $p = 0.950$ might be a more likely value.[11]

In Table 1 we present optimal replacement rates calculated for the whole range of values that go from no price drop ($p = 1.000$), through our preferred estimate of a 5% price drop ($p = 0.950$), to a 10% price drop ($p = 0.900$). This last value implies that the fall in expenditure is entirely due to the price change.

The first column of Table 1 shows the optimal replacement ratio as a function of relative risk-aversion in the case in which the distinction between consumption and expenditure is irrelevant ($p = 1$). The values in this column are approximately equal to those of Gruber (1997, p. 203, Table 4, Col. 1). The difference with Gruber's values is explained by our use of the exact expression in Eq. (15) rather than the linear approximation, and by our choice of a slightly larger $\varepsilon_{D,b}$.[12]

As we move to the columns on the right, increasing the gap between consumption and expenditure, optimal replacement ratios decrease and it becomes more difficult to reconcile the replacement rates with those prevailing in the US assuming reasonable values of relative risk-aversion. For example, for our preferred estimate of $p = 0.950$ it takes a value of γ of almost 5 to reach an optimal replacement rate of 40%, close to replacement rates in the data.

[8] Recent examples of studies using Gruber's approach include Browning and Crossley (2001) and Bronchetti (2012).

[9] In a recent survey, Tatsiramos and van Ours (2014) report an average replacement rate of 53% for the US. Coupled with estimates obtained by Gruber (for a prior period), this replacement rate implies an 8% drop in consumption expenditure upon unemployment.

[10] In the Appendix we re-calculate our results for $\varepsilon_{D,b} = 0.432$, the value preferred by Gruber (1997), using the linear approximation in Eq. (16).

[11] Aguiar and Hurst (2005, pp. 943–944) make a comment that goes in this direction when they compare their consumption index to the 9% drop in earnings six or more years after the onset of unemployment uncovered by Stevens (1997). Their reason for confronting their index with long-term earnings is that the consumption index is expressed in terms of permanent income dollars.

[12] In the Appendix we show that Gruber's results are exactly replicated once these changes are undone.

Table 1
Optimal replacement ratio.

Level of RRA	Relative price ratio p				
γ	1.000	0.975	0.950	0.925	0.900
1.0	0.000	0.000	0.000	0.000	0.000
1.5	0.000	0.000	0.000	0.000	0.000
2.0	0.160	0.112	0.063	0.013	0.000
2.5	0.296	0.238	0.180	0.119	0.057
3.0	0.386	0.322	0.257	0.190	0.121
3.5	0.451	0.382	0.312	0.240	0.167
4.0	0.499	0.427	0.354	0.278	0.201
5.0	0.567	0.490	0.412	0.331	0.249

Table 2
Optimal replacement ratio: deviations from a canonical Baily–Chetty formula.

Level of RRA	Relative price ratio p				
γ	1.000	0.975	0.950	0.925	0.900
1.0	0.000	0.000	0.000	0.000	0.000
1.5	0.000	−0.032	−0.065	−0.098	−0.133
2.0	0.000	−0.048	−0.097	−0.147	−0.199
2.5	0.000	−0.057	−0.116	−0.177	−0.239
3.0	0.000	−0.064	−0.129	−0.196	−0.265
3.5	0.000	−0.068	−0.138	−0.210	−0.284
4.0	0.000	−0.072	−0.145	−0.221	−0.298
5.0	0.000	−0.076	−0.155	−0.235	−0.318

3.1. The consumption–expenditure wedge

How exactly is the optimal replacement rate impacted by the distinction between consumption and expenditure? From Eq. (18), the optimal replacement ratio can be written as

$$r^*(\gamma, p) = r^*(\gamma, 1) + \frac{1-\gamma}{\gamma\hat{\beta}} \log p. \tag{19}$$

The value $r^*(\gamma, 1)$ corresponds to the replacement ratio if the distinction between consumption and expenditure were neglected (this is equivalent to assuming $p = 1$) whereas $r^*(\gamma, p)$ is the correct replacement ratio. We define the consumption–expenditure wedge *CEW* as

$$CEW(\gamma, p) \equiv r^*(\gamma, p) - r^*(\gamma, 1) = \frac{1-\gamma}{\gamma\hat{\beta}} \log p. \tag{20}$$

Focusing on the wedge *CEW* is informative for a wider class of models that extend the original Baily–Chetty setup. These models lead to versions of Eq. (14) that include small variations but are overall similar to it. For example, Kroft (2008) analyzes the optimal unemployment insurance benefit level in a model with imperfect takeup and obtains the original Baily–Chetty formula with an added social multiplier term. The standard formula plus an extra term involving the ratio of private to public insurance is obtained in the model of Chetty and Saez (2010), who consider the case of endogenous private insurance in an environment in which expanding public insurance has a fiscal externality on private insurers. In the context of health insurance, the standard Baily–Chetty approximation plus an additive term is obtained if marginal utility is affected by health (Finkelstein et al., 2013; Chetty and Finkelstein, 2013).[13] Because p shows up in the optimization problem multiplying the marginal utility of consumption when unemployed, it is bound to show up in a way similar to Eq. (14) in generalizations of the standard Baily–Chetty formula.

The *CEW* is solely a function of p and parameters γ and $\hat{\beta}$. This dependence on a reduced set of parameters is interesting also from a purely empirical perspective. The calculations in Table 1 are influenced by variables that have only level effects and do therefore not show up in the wedge. The duration elasticity $\varepsilon_{D,b}$ is one such variable. The other one is α. Whereas there exists some degree of consensus on the value of $\varepsilon_{D,b}$, this is not the case with α.[14]

The parameter α measures the drop in expenditure that workers in the US would face in the absence of unemployment insurance. Of course, this is a counter-factual experiment that is never observed in real life. The estimated value of α is an extrapolation that owes its existence to the linear specification that was originally assumed by Gruber (1997), and which is reflected in Eq. (17). The slope parameter $\hat{\beta}$, on the other hand, is identified primarily from local variation inside the range of values that are observed in the data.

By focusing on the *CEW* we strip the empirical implications of our model from the level effects that are not directly relevant to study the impact of generalizing the standard model. In Table 2 we show how the wedge depends on the choices of the relative risk-aversion parameter and the value of p.[15] The values in this table show how much the optimal replacement rate needs to be reduced if the distinction between consumption and expenditure is neglected as in the standard Baily–Chetty formula.[16]

The values in this table confirm that the wedge is very sensitive to p. From the definition of CEW in Eq. (20), for $\gamma > 1$ the wedge is always negative. The logarithmic function implies that departures of the price p away from 1 lead to an effect on the wedge of increasing magnitude. This high sensitivity to p highlights the need of having estimates of the parameter p as precise as possible to calibrate optimal replacement rates.

4. Discussion of assumptions and extensions

4.1. Preferences

The characterization of optimal unemployment insurance in Proposition 1 is expressed in terms of marginal utility and does therefore not require any specific assumption on preferences. In contrast, for the characterization in terms of observational data in Proposition 2 we assumed CRRA preferences. With CRRA preferences, because marginal utility is proportional to the inverse of consumption raised to a power, it is straightforward to collect price and quantity terms. Other preferences do not have this useful property.

Also, in the case of CRRA preferences, whether the standard Baily–Chetty model overstates or understates the level of optimal unemployment insurance depends on a single parameter: the degree of relative risk aversion γ. With less parsimonious utility functions, the result will depend on a bigger set of parameters. Intuitively, the answer to the question requires combining the willingness to substitute across states of the world with how a consumer assigns expenditure shares. In the CRRA case, these two aspects of the problem are nicely captured by the coefficient of relative risk-aversion.

It may be possible to extend the results in Proposition 2 to more general preferences. In particular, if preferences can be approximated by a CRRA function, then the results will also hold approximately. For other

[13] A complete survey of papers that extend the Baily–Chetty formula is beyond the scope of our paper. Several of these studies are surveyed by Chetty and Finkelstein (2013 Section 3).

[14] There is still some uncertainty surrounding the correct value of $\varepsilon_{D,b}$. For example, some calibrations use values close to 0.75 whereas recent research by Meyer and Mok (2014) suggests values in the range 0.1–0.2.

[15] As pointed out by an anonymous referee, it may be that γ and p are themselves dependent on α and $\varepsilon_{D,b}$ if, for example, low levels of consumption for the unemployed in the absence of insurance generate more risk aversion (Chetty and Szeidl, 2007) or greater time directed to shopping at the optimum. The results in Table 2 should therefore be considered bearing this caveat in mind.

[16] The values in Table 2 cannot be directly derived from those in Table 1 because of the presence of corner solutions for combinations of low risk aversion levels and large relative price drops.

Table 3
Optimal replacement ratio using the linear approximation and same parameter values as Gruber (1997).

Level of RRA	Relative price ratio p				
γ	1.000	0.975	0.950	0.925	0.900
1.0	0.000	0.000	0.000	0.000	0.000
1.5	0.000	0.000	0.000	0.000	0.000
2.0	0.023	0.000	0.000	0.000	0.000
2.5	0.186	0.129	0.072	0.016	0.000
3.0	0.294	0.231	0.169	0.106	0.043
3.5	0.372	0.305	0.237	0.170	0.102
4.0	0.430	0.359	0.289	0.218	0.147
5.0	0.512	0.436	0.361	0.285	0.210

preferences, the mapping from the characterization of optimal unemployment insurance in Proposition 1 to observational data needs to be derived on a case-by-case basis.

4.2. Prices and quality

The results by Aguiar and Hurst (2007) show an association between higher shopping effort and price drops holding quality constant (because they used scanner data and studied individual UPC codes). This motivated our focus on price drops rather than changes in quality.

It is possible, and there is evidence (e.g., Banks et al., 1998), that exiting the labor force leads to substitution in consumption categories in the direction of lower quality products. Therefore, an interesting avenue for future research is to incorporate quality as a choice variable, ideally in combination with the decision to shop for lower prices, as in our model.

Although it is hard to speculate on how optimal unemployment insurance would be affected in such a model, a reasonable prediction is that workers would react to unemployment by choosing a combination of lower prices and lower quality. If so, at the margin, the value of leisure time lost to increased price search should equal the disutility produced by switching to a lower quality product. As is usual in the "sufficient statistics" approach, this means that, even in this more general setting, observing price drops might suffice to back out the effect of the (possibly unobservable) reduction in quality.

4.3. Externalities

A general result in the "sufficient statistics" literature is that the formulas derived continue to hold irrespective of the structure of the model if agents' choices in the private sector maximize private surplus. Choices do not maximize private surplus if there are externalities among agents in the private sector.

In our case, lower prices paid by the unemployed imply lower prices charged by the sellers of consumption goods. Because we assumed a zero-profit condition, in our model there is no negative externality to be concerned about. To study how the results in the model change if we lifted the zero-profit condition, in Appendix B we study a model that explicitly takes into account the presence of sellers whose profits are affected by the choices of workers. We summarize the findings for this model with sellers here and refer to the Appendix for details.

The analysis from this model opens the door to lower optimal unemployment benefits than those in our benchmark model because of a profit externality: unemployed workers do not take into account how the time spent on job search and price search impacts the profits and welfare of sellers; thus, unemployed workers may spend too much time on price search and too little time on job search from what is socially optimal when the welfare of sellers is explicitly taken into account.

We find that the exact relationship between the worker's choices and optimal unemployment benefits hinges on the magnitude of two elasticities. In addition to the elasticity of the duration of unemployment with respect to unemployment benefits $\varepsilon_{D,b}$, the elasticity of prices with respect to unemployment benefits, which we term $\varepsilon_{p,b}$, becomes an important input for the analysis. We are not aware of any studies that estimate the price elasticity $\varepsilon_{p,b}$ for the US, or other countries. Because this elasticity informs economic theory, attempts to estimate the price elasticity $\varepsilon_{p,b}$ might be a fruitful direction for future empirical research.

From analyzing the extended model with sellers, we conclude that, although definite results depend on the sign and the magnitude of the price elasticity $\varepsilon_{p,b}$, the profit externality that affects the welfare of sellers tends to push optimal unemployment benefits lower when compared to the model in Section 2.

5. Concluding remarks

We have shown how the provision of optimal unemployment insurance is affected by the distinction between consumption and expenditure. By extending the Baily–Chetty model we derived a "sufficient statistics" formula for optimal unemployment insurance that is expressed in terms of observable variables and can therefore be readily used in applied work.

For levels of relative risk-aversion that exceed one, our formula indicates that optimal unemployment insurance benefits are lower when the distinction between consumption and expenditure is taken into account. A calibration to US data suggests that the effect of this distinction on the magnitude of optimal replacement rates is quantitatively large.

There is abundant evidence on the magnitude of expenditure drops at unemployment for the US and for several other countries. In comparison, the evidence on price changes upon unemployment is still relatively scarce, with the possible exception of the US. Still, even in the case of the US, important research lies ahead. The evidence so far is for a reduced number of items (typically groceries) and is based on cross-sectional data. To calibrate the value of p, ideally, one would want to use panel-data estimates of prices paid on a wide range of goods by households transitioning into unemployment.[17]

How much consumption-smoothing is achieved is an important attribute of any unemployment insurance scheme. In this paper we have shown that to correctly quantify the benefits of consumption-smoothing it is necessary to distinguish between consumption and expenditure. Because many social insurance programs have consumption insurance at their heart, this distinction will be relevant not only for the design of optimal unemployment insurance, but for social insurance programs in general.

Appendix A. Results for the linear approximation

Because the results by Gruber (1997) are frequently used in applied work, we calculate comparable optimal replacement ratios using the linear approximation and parameter values assumed by Gruber (1997). In his calculations, Gruber (1997) assumed a duration elasticity $\varepsilon_{D,b}$ of 0.432 and used the linear approximation in Eq. (16). Table 3 exhibits optimal replacement ratios that are comparable to those in Table 4 of Gruber (1997).

The values in the first column, in which $p = 1$, replicate the values in the column denoted as the 'Base Case' in Table 4 of Gruber (1997). In comparison with Table 1, optimal replacement ratios are slightly lower than in Table 1 despite of the use of a slightly lower duration elasticity. The reason for this is the linear approximation error: in the approximation, Gruber (1997) sets $\varepsilon_{D,b} = 0.432$ whereas the exact formula is effectively using $\log(1 + 0.500) = 0.405$.

[17] Panel data that could be fruitfully employed for this purpose exists for countries other than the US, for example for Spain (Luengo-Prado and Sevilla, 2013; Campos and Reggio, 2015).

Appendix B. An extended model with sellers

In this section we extend our model to include a group of sellers.[18]

The model with sellers

Sellers are individuals who simply receive utility from the profits they obtain from selling consumption goods to the employed and unemployed individuals in the model. Including sellers in the model, total welfare is equal to:

$$W = V + V_S, \tag{21}$$

where V is the welfare of workers, as before, and $V_S = u_S(c_S)$ is the utility received by the sellers from consumption c_S. Suppose that sellers pay an economic cost of $z < p$ for each unit of consumption good. Then their consumption is

$$c_S = (1-\pi)(1-z)c_e + \pi(p-z)c_u. \tag{22}$$

Substituting from the worker's two budget constraints, Eqs. (4) and (5), and the government balanced budget condition (1), this becomes

$$\begin{aligned} c_S &= (1-\pi)(1-z)\left[A + w - \frac{\pi}{1-\pi}bD\right] + \pi\left(\frac{p-z}{p}\right)[A + bD + w(1-D)] \\ &= \bar{c}_S + \pi\left\{\left(\frac{p-z}{p}\right)[A + w-(w-b)D]-(1-z)bD\right\}, \end{aligned} \tag{23}$$

where $\bar{c}_S = (1-\pi)(1-z)(A+w)$ consists of elements that we have grouped together because they do not depend on the endogenous variables b, p, or D.

Using the derivative of total welfare with respect to b:

$$\frac{dW}{db} = \frac{dV}{db} + \left(\frac{\partial V_S}{\partial b} + \frac{\partial V_S}{\partial p}\frac{dp}{db} + \frac{\partial V_S}{\partial D}\frac{dD}{db}\right), \tag{24}$$

the marginal net benefit of a balanced-budget increase of b can be expressed in terms of the duration elasticity $\varepsilon_{D,b}$ and a price elasticity $\varepsilon_{p,b}$, which we define below.

Proposition 3. *In a model with sellers, the marginal net benefit of a balanced-budget increase of b is given by*

$$\begin{aligned} \frac{dW}{db} &= \frac{dV}{db} + \pi D u_S'(\cdot) \\ &\times \left[z\left(-\frac{1-p}{p}(1+\varepsilon_{D,b}) + \frac{c_u}{bD}\varepsilon_{p,b}\right) - \left(\frac{p-z}{p}\right)\frac{w}{b}\varepsilon_{D,b}\right], \end{aligned} \tag{25}$$

where $\varepsilon_{D,b}$ is the elasticity of the duration of unemployment with respect to the unemployment insurance benefit b defined in Eq. (13) and $\varepsilon_{p,b}$ is the elasticity of prices paid by the unemployed with respect to the unemployment insurance benefit b:

$$\varepsilon_{p,b} = \frac{b}{p}\frac{dp}{db}. \tag{26}$$

Proof: in Appendix C.

Discussion

The price elasticity $\varepsilon_{p,b}$ is theoretically ambiguous: the higher is b, the less need unemployed individuals have to search for lower prices, but the longer they will spend unemployed and thus the more time they have available for price search. This implies that, although the other terms inside the brackets are clearly negative, the sign of the expression inside the brackets in Eq. (25) depends on the sign and magnitude of the price elasticity $\varepsilon_{p,b}$.

If the price elasticity $\varepsilon_{p,b}$ is negative, then optimal unemployment benefits are unequivocally lower when compared to the model that does not take into account sellers. The basic intuition is that, given the profit externality, unemployed individuals exert too little job search effort (because being unemployed reduces expenditure and thus profits), but too much price search effort (because they do not account for the losses to the sellers), and reducing b leads to a reallocation of time into job search. Of course, whether the price elasticity $\varepsilon_{p,b}$ is negative in practice is a question that must be settled empirically.

Some configurations of parameters lead to lower unemployment benefits regardless of the sign of $\varepsilon_{p,b}$. In this respect, the economic cost z that sellers pay for each consumption good plays an important role. Inspecting Eq. (25), it is apparent that for small enough z, the sign of the term in brackets is unambiguously negative, thus leading lower optimal unemployment benefits.[19] For example, in the case in which the economic cost $z = 0$, consumption goods are costless and profits coincide with revenue. In this special case, lower prices paid by the unemployed do not harm sellers because they are exactly counteracted by the larger quantities bought by the unemployed (which are free for the sellers). Therefore, the welfare of the sellers does not depend on prices. In this case, lower unemployment benefits are optimal because they counteract a pure revenue externality: reducing b reduces unemployment duration, and therefore increases income that can be spent on consumption goods, and provide utility to sellers.

Attempting to obtain more definite results by specifying the precise value of z goes beyond the purpose of this section. However, we note that low values of z are not empirically implausible because, in this version of the model, sellers embed all stages of production prior to the final sale of consumption goods. From the extended model with sellers we conclude that, if the zero-profit condition is lifted, there are forces that lead to lower optimal unemployment benefits because of a profit externality. We also show that, for certain parameterizations, optimal unemployment benefits are strictly lower in this case.

As a final point, we note that whether the inclusion of sellers has a quantitatively large effect on optimal unemployment insurance depends on the magnitude of marginal utility $u'(c_S)$, which scales the term in brackets in (25).[20] If profits go to very wealthy individuals, whose marginal utility is so low as to be basically zero, then the externality on sellers can be ignored by the planner as it will have no effect on optimal unemployment insurance.

Appendix C. Proofs

Proof of Proposition 1. The indirect utility function that the planner maximizes can be written fully in terms of b. Using the λ_e and λ_u to denote the Lagrange multipliers of the two budget constraints in the worker's problem, the indirect utility function $V(b, \tau(b))$ is equal to

$$\begin{aligned} V(b,\tau(b)) = \max_{c_e,c_u,D,p,\lambda_e,\lambda_u,\{\lambda_0^i,\lambda_1^i\}_{i=D,p,\ell}} & (1-\pi)u(c_e) + \pi\{u(c_u) + \psi(D,p)\} \\ & + \lambda_e[A + (w-\tau(b))-c_e] + \lambda_u[A + bD + w(1-D)-pc_u] \\ & + \lambda_0^D D + \lambda_1^D(1-D) + \lambda_0^p\left(p-\underline{p}\right) + \lambda_1^p(1-p) + \lambda_0^\ell \ell(D,p) \\ & + \lambda_1^\ell(1-\ell(D,p)). \end{aligned} \tag{27}$$

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[18] We thank an anonymous referee for suggesting the inclusion of a model with sellers and guiding our modeling choices in this section.

[19] Stated formally, for any $\varepsilon_{D,b} > 0$ and $\varepsilon_{p,b} < \infty$ there exists a real number $\bar{z} > 0$ such that, if $z < \bar{z}$, then the term in brackets in Eq. (25) is negative. This result follows directly from evaluating Eq. (25) at $z = 0$ and using the fact that the expression in brackets in Eq. (25) is a continuous function of z.

[20] Although π and D also scale the term in brackets, they do not have a differential impact. Notice that they also appear in Eq. (12), the expression describing the marginal benefit to workers as well.

Because this expression is optimized over $\{c_e, c_u, D, p, \lambda_e, \lambda_u, \lambda_0^D, \lambda_1^D, \lambda_0^p, \lambda_1^p, \lambda_0^\ell, \lambda_1^\ell\}$, by the Envelope Theorem, changes in these variables do not have first-order effects on $V(b, \tau(b))$. Therefore,

$$\frac{dV(b,\tau(b))}{db} = -\lambda_e \frac{d\tau(b)}{db} + \lambda_u D. \tag{28}$$

Agent optimization implies

$$\lambda_e = (1-\pi)u'(c_e) \tag{29}$$

and

$$p\lambda_u = \pi u'(c_u). \tag{30}$$

Finally, differentiating the government's budget constraint,

$$\frac{d\tau(b)}{db} = \frac{\pi}{1-\pi}\left[D + b\frac{dD}{db}\right]. \tag{31}$$

Substitute λ_e, λ_u, and the derivative $\frac{d\tau(b)}{db}$ in Eq. (28) using the first order conditions (29) and (30), and (31). After these substitutions, and rearranging, Eq. (28) becomes

$$\frac{dV(b)}{db} = \pi D\left[\frac{1}{p}u'(c_u) - \left[1 + \frac{b}{D}\frac{dD}{db}\right]u'(c_e)\right]. \tag{32}$$

Applying the definition of the elasticity, the exact expression in the Proposition is obtained. Q.E.D.

Proof of Proposition 2. Substitute $u'(c) = c^{-\gamma}$ in Eq. (14) and take logs on both sides to obtain

$$\gamma \log c_e - \gamma \log c_u = \log(1 + \varepsilon_{D,b}) + \log p. \tag{33}$$

Subtract $\gamma \log p$ on both sides. On the left hand side of this equation, note that $\gamma \log c_e = \gamma \log \tilde{c}_e$ and $\gamma \log p + \gamma \log c_u = \gamma \log(pc_u) = \gamma \log \tilde{c}_u$. Make these substitutions and factor out γ on the left hand side and collect terms involving $\log p$ on the right hand side of the equation to arrive at Eq. (15).

To obtain the approximation start from Eq. (15) and on the left hand side write $\Delta \log \tilde{c} = \log(1 + \frac{\Delta \tilde{c}}{\tilde{c}})$. Then, on both sides of the equation, use the approximation $\log(1 + z) \approx z$. This yields Eq. (16). Q.E.D.

Proof of Proposition 3. The proof consists of calculating the derivatives inside the parentheses in Eq. (24). The first term inside the parentheses is

$$\frac{\partial V_S}{\partial b} = \pi D u_s'(\cdot) z\left(1 - \frac{1}{p}\right). \tag{34}$$

The second term is

$$\begin{aligned}\frac{\partial V_S}{\partial p}\frac{dp}{db} &= u_s'(\cdot)\pi[A + w - (w-b)D]\frac{z}{p^2}\frac{dp}{db} \\ &= \pi D u_s'(\cdot) z \frac{c_u}{bD}\varepsilon_{p,b}.\end{aligned} \tag{35}$$

The third term is

$$\begin{aligned}\frac{\partial V_S}{\partial D}\frac{dD}{db} &= u_s'(\cdot)\pi\left[\left(\frac{p-z}{p}\right)(b-w) - (1-z)b\right]\frac{dD}{db} \\ &= \pi D u_s'(\cdot)\left[\left(\frac{p-z}{p}\right)\left(1 - \frac{w}{b}\right) - (1-z)\right]\varepsilon_{D,b}.\end{aligned} \tag{36}$$

Substitute the three terms into Eq. (24) to obtain Eq. (25) after some algebra. Q.E.D.

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