Online Appendix
Moral Hazard versus Liquidity and the Optimal Timing of Unemployment Benefits

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A Theoretical appendix

A.1 Proofs

Lemma 1. The first order condition of the problem of an unemployed agent with respect to search effort $s_{i,t+1}(\omega_{i,t})$ is, for all $\omega_{i,t}$:

$$-\frac{\partial v_i^u(c_{i,t}^u(\omega_{i,t}), s_{i,t+1}(\omega_{i,t}))}{\partial s_{i,t+1}(\omega_{i,t})} = \beta\left[\mathbb{E}_t V_{i,t+1}^e(\omega_{i,t+1}) - \mathbb{E}_t V_{i,t+1}^u(\omega_{i,t+1})\right] h_{i,t+1}'(s_{i,t+1}(\omega_{i,t})). \tag{1}$$

For $t = 0$, this first order condition is

$$-\frac{\partial v_i^u(c_{i,0}^u(\omega_{i,0}), s_{i,1}(\omega_{i,0}))}{\partial s_{i,1}(\omega_{i,0})} = \beta\left[\mathbb{E}_0 V_{i,1}^e(\omega_{i,1}) - \mathbb{E}_0 V_{i,1}^u(\omega_{i,1})\right] h_{i,1}'(s_{i,1}(\omega_{i,0})). \tag{2}$$

Notice that the expectations in the expression are conditional on information available at date $t = 0$ and are therefore functions of $\omega_{i,0}$, the exogenous initial condition. Taking the derivative of the first order condition with respect to $x \in \{b_j, w_j, y_j\}$, $j \geq 1$, produces:

$$\begin{aligned}
-\frac{\partial^2 v_i^u(\cdot)}{\partial s \partial c^u}\frac{\partial c_{i,0}^u(\omega_{i,0})}{\partial x} &- \frac{\partial^2 v_i^u(\cdot)}{\partial s^2}\frac{\partial s_{i,1}(\omega_{i,0})}{\partial x} \\
&= \beta\left[\frac{\partial}{\partial x}\mathbb{E}_0 V_{i,1}^e(\omega_{i,1}) - \frac{\partial}{\partial x}\mathbb{E}_0 V_{i,1}^u(\omega_{i,1})\right] h_{i,1}'(s_{i,1}(\omega_{i,0})) \\
&+ \beta\left[\mathbb{E}_0 V_{i,1}^e(\omega_{i,1}) - \mathbb{E}_0 V_{i,1}^u(\omega_{i,1})\right] h_{i,1}''(s_{i,1}(\omega_{i,0}))\frac{\partial s_{i,1}(\omega_{i,0}))}{\partial x}.
\end{aligned} \tag{3}$$

This expression can be rearranged as

$$\frac{\partial s_{i,1}(\omega_{i,0})}{\partial x} = \Lambda(\omega_{i,0})\left[\frac{\partial}{\partial x}\mathbb{E}_0 V_{i,1}^e(\omega_{i,1}) - \frac{\partial}{\partial x}\mathbb{E}_0 V_{i,1}^u(\omega_{i,1}) + \frac{\partial^2 v_i^u(\cdot)}{\partial s \partial c^u}\frac{\partial c_{i,0}^u(\omega_{i,0})}{\partial x}\right], \tag{4}$$

where

$$\Lambda(\omega_{i,0}) \equiv \frac{\beta h_{i,1}'(s_{i,1}(\omega_{i,0}))}{-\frac{\partial^2 v_i^u(\cdot)}{\partial s^2} + \frac{\partial v_i^u(\cdot)}{\partial s}\frac{h_{i,1}''(s_{i,1}(\omega_{i,0}))}{h_{i,1}'(s_{i,1}(\omega_{i,0}))}} > 0. \tag{5}$$

If the utility function is separable in consumption and search effort, then (4) simplifies to:

$$\frac{\partial s_{i,1}(\omega_{i,0})}{\partial x} = \Lambda(\omega_{i,0})\left[\frac{\partial}{\partial x}\mathbb{E}_0 V_{i,1}^e(\omega_{i,1}) - \frac{\partial}{\partial x}\mathbb{E}_0 V_{i,1}^u(\omega_{i,1})\right]. \tag{6}$$

However, we do not impose the assumption of separable utility in this Lemma.

Using the Envelope condition for the maximized functions V^e and V^u, the effect on the expected value functions in period t of raising benefits, wages, or non-labor income in period $t+j$, $j \geq 1$, for any fixed $\omega_{i,t}$ is:

$$\begin{aligned}
\frac{\partial \mathbb{E}_t V^e_{i,t+1}(\omega_{i,t+1})}{\partial b_{t+j}} &= 0, \quad j \geq 1 \\
\frac{\partial \mathbb{E}_t V^u_{i,t+1}(\omega_{i,t+1})}{\partial b_{t+j}} &= \beta^j S_{i,t+j} \mathbb{E}_t \left[v^u_{i,1}(c^u_{i,t+j}(\omega_{i,t+j}), s_{i,t+j+1}(\omega_{i,t+j})) | U \right], \quad j \geq 1
\end{aligned} \tag{7}$$

$$\begin{aligned}
\frac{\partial \mathbb{E}_t V^e_{i,t+1}(\omega_{i,t+1})}{\partial w_{t+j}} &= \beta^j \mathbb{E}_t \left[v^e_{i,1}(c^e_{i,t+j}(\omega_{i,t+j})) | E \right], \quad j \geq 1 \\
\frac{\partial \mathbb{E}_t V^u_{i,t+1}(\omega_{i,t+1})}{\partial w_{t+j}} &= \beta^j (1 - S_{i,t+j}) \mathbb{E}_t \left[v^e_{i,1}(c^e_{i,t+j}(\omega_{i,t+j})) | E \right], \quad j \geq 1
\end{aligned} \tag{8}$$

$$\begin{aligned}
\frac{\partial \mathbb{E}_t V^e_{i,t+1}(\omega_{i,t+1})}{\partial y_{t+j}} &= \beta^j \mathbb{E}_t \left[v^e_{i,1}(c^e_{i,t+j}(\omega_{i,t+j})) | E \right], \quad j \geq 1 \\
\frac{\partial \mathbb{E}_t V^u_{i,t+1}(\omega_{i,t+1})}{\partial y_{t+j}} &= \beta^j (1 - S_{i,t+j}) \mathbb{E}_t \left[v^e_{i,1}(c^e_{i,t+j}(\omega_{i,t+j})) | E \right] \\
&\quad + \beta^j S_{i,t+j} \mathbb{E}_t \left[v^u_{i,1}(c^u_{i,t+j}(\omega_{i,t+j}), s_{i,t+j+1}(\omega_{i,t+j})) | U \right], \quad j \geq 1
\end{aligned} \tag{9}$$

For $t = 0$, these results imply that, for all $j \geq 1$,

$$\frac{\partial \mathbb{E}_0 V^e_{i,1}(\omega_{i,1})}{\partial b_j} = \frac{\partial \mathbb{E}_0 V^e_{i,1}(\omega_{i,1})}{\partial y_j} - \frac{\partial \mathbb{E}_0 V^e_{i,1}(\omega_{i,1})}{\partial w_j} \tag{10}$$

and

$$\frac{\partial \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})}{\partial b_j} = \frac{\partial \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})}{\partial y_j} - \frac{\partial \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})}{\partial w_j}, \tag{11}$$

and, after subtracting these two equations,

$$\begin{aligned}
\left(\frac{\partial \mathbb{E}_0 V^e_{i,1}(\omega_{i,1})}{\partial b_j} - \frac{\partial \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})}{\partial b_j} \right) &= \left(\frac{\partial \mathbb{E}_0 V^e_{i,1}(\omega_{i,1})}{\partial y_j} - \frac{\partial \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})}{\partial y_j} \right) \\
&\quad - \left(\frac{\partial \mathbb{E}_0 V^e_{i,1}(\omega_{i,1})}{\partial w_j} - \frac{\partial \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})}{\partial w_j} \right).
\end{aligned} \tag{12}$$

Inspection of the agent's problem reveals that y_j, b_j, w_j only appear in budget constraints and that an increase in y_j compensated by a simultaneous decrease in b_j and w_j leaves all budget constraints unchanged. Therefore, optimal consumption must satisfy:

$$\frac{\partial c^u_{i,0}(\omega_{i,0})}{\partial y_j} - \frac{\partial c^u_{i,0}(\omega_{i,0})}{\partial b_j} - \frac{\partial c^u_{i,0}(\omega_{i,0})}{\partial w_j} = 0. \tag{13}$$

Using this fact, (12) can also be expressed in the following way (by adding a term that equals zero to the equation):

$$\begin{aligned}
&\left(\frac{\partial \mathbb{E}_0 V^e_{i,1}(\omega_{i,1})}{\partial b_j} - \frac{\partial \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})}{\partial b_j} + \frac{\partial^2 v^u_i(\cdot)}{\partial s \partial c^u}\frac{\partial c^u_{i,0}(\omega_{i,0})}{\partial b_j}\right) \\
&\quad = \left(\frac{\partial \mathbb{E}_0 V^e_{i,1}(\omega_{i,1})}{\partial y_j} - \frac{\partial \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})}{\partial y_j} + \frac{\partial^2 v^u_i(\cdot)}{\partial s \partial c^u}\frac{\partial c^u_{i,0}(\omega_{i,0})}{\partial y_j}\right) \\
&\quad\quad - \left(\frac{\partial \mathbb{E}_0 V^e_{i,1}(\omega_{i,1})}{\partial w_j} - \frac{\partial \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})}{\partial w_j} + \frac{\partial^2 v^u_i(\cdot)}{\partial s \partial c^u}\frac{\partial c^u_{i,0}(\omega_{i,0})}{\partial w_j}\right). \qquad (14)
\end{aligned}$$

By multiplying both sides by $\Lambda(\omega_{i,0}) \neq 0$, and comparing with (4), it follows that

$$\frac{\partial s_{i,1}(\omega_{i,0})}{\partial b_j} = \frac{\partial s_{i,1}(\omega_{i,0})}{\partial y_j} - \frac{\partial s_{i,1}(\omega_{i,0})}{\partial w_j}, \quad j \geq 1. \qquad (15)$$

The final step is to multiply both sides of this equation by $h'_{i,1}(s_{i,1}(\omega_{i,0})) \neq 0$ and to notice that, by the chain-rule,

$$\frac{\partial h_{i,1}(s_{i,1}(\omega_{i,0}))}{\partial x} = h'_{i,1}(s_{i,1}(\omega_{i,0}))\frac{\partial s_{i,1}(\omega_{i,0})}{\partial x},$$

for variables $x \in \{b_j, y_j, w_j\}$. This leads to the expression in the Lemma. Q.E.D.

Lemma 2. From the expressions derived in the proof of Lemma 1, for separable utility:

$$\begin{aligned}
\frac{1}{\Lambda(\omega_{i,t})}\frac{\partial s_{i,t+1}(\omega_{i,t})}{\partial y_{t+j}} &= \frac{\partial}{\partial y_{t+j}}\mathbb{E}_t V^e_{i,t+1}(\omega_{t+1}) - \frac{\partial}{\partial y_{t+j}}\mathbb{E}_t V^u_{i,t+1}(\omega_{t+1}) \\
&= \beta^j S_{i,t+j}\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j}(\omega_{i,t+j}))|E\right] \\
&\quad - \beta^j S_{i,t+j}\mathbb{E}_t\left[v^u_{i,1}(c^u_{i,t+j}(\omega_{i,t+j}), s_{i,t+j+1}(\omega_{i,t+j}))|U\right]. \qquad (16)
\end{aligned}$$

Taking the ratio of this equation evaluated in two consecutive periods $t+j$ and $t+j+1$ yields:

$$\frac{\frac{\partial s_{i,t+1}}{\partial y_{t+j+1}}}{\frac{\partial s_{i,t+1}}{\partial y_{t+j}}} = \beta\frac{S_{i,t+j+1}}{S_{i,t+j}}\frac{\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j+1})|E\right] - \mathbb{E}_t\left[v^u_{i,1}(c^u_{i,t+j+1}, s_{i,t+j+2})|U\right]}{\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j})|E\right] - \mathbb{E}_t\left[v^u_{i,1}(c^u_{i,t+j}, s_{i,t+j+1})|U\right]} \qquad (17)$$

When the borrowing constraint does not bind, the Euler equation of an unemployed worker between two consecutive periods is

$$\mathbb{E}_t v^e_{i,1}(c_{i,t+j}) = (1+r)\beta\mathbb{E}_t v^e_{i,1}(c_{i,t+j+1}) \qquad (18)$$

and the Euler equation of an unemployed worker is

$$\mathbb{E}_t v^u_{i,1}(c_{i,t+j},\cdot) = (1+r)\beta[(1-h_{i,t+j+1})\mathbb{E}_t v^u_{i,1}(c_{i,t+j+1},\cdot) + h_{i,t+j+1}\mathbb{E}_t v^e_{i,1}(c_{i,t+j+1},\cdot)]. \qquad (19)$$

Subtracting these two Euler equations and rearranging yields:

$$\frac{\mathbb{E}_t v^e_{i,1}(c_{i,t+j+1}) - \mathbb{E}_t v^u_{i,1}(c_{i,t+j+1},\cdot)}{\mathbb{E}_t v^e_{i,1}(c_{i,t+j}) - \mathbb{E}_t v^u_{i,1}(c_{i,t+j},\cdot)} = \frac{1}{(1+r)\beta(1-h_{i,t+j+1})} \qquad (20)$$

Substituting this expression into (17) simplifies to

$$\frac{\frac{\partial s_{i,t+1}}{\partial y_{t+j+1}}}{\frac{\partial s_{i,t+1}}{\partial y_{t+j}}} = \frac{S_{i,t+j+1}}{S_{i,t+j}(1-h_{i,t+j+1})}\frac{1}{1+r} = \frac{1}{1+r} \tag{21}$$

This implies that

$$\frac{\partial s_{i,t+1}}{\partial y_{t+j+1}} = \frac{\partial s_{i,t+1}}{\partial y_{t+1}}(1+r)^{-j} \tag{22}$$

Multiplying both sides by $h'(s_{t+1}) \neq 0$ and noticing that, by the Chain rule, $\frac{\partial h_{i,t+1}}{\partial x} = h'(s_{t+1})\frac{\partial s_{i,t+1}}{\partial x}$ produces:

$$\frac{\partial h_{i,t+1}}{\partial y_{t+j+1}} = \frac{\partial h_{i,t+1}}{\partial y_{t+1}}(1+r)^{-j}. \tag{23}$$

This is the first equation in the Lemma.

The impact of the wage rate on search effort is given by the following relationship:

$$\begin{aligned}\frac{1}{\Lambda(\omega_{i,t})}\frac{\partial s_{i,t+1}(\omega_{i,t})}{\partial w_{t+j}} &= \frac{\partial}{\partial w_{t+j}}\mathbb{E}_t V^e_{i,t+1}(\omega_{t+1}) - \frac{\partial}{\partial w_{t+j}}\mathbb{E}_t V^u_{i,t+1}(\omega_{t+1}) \\ &= \beta^j S_{i,t+j}\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j}(\omega_{i,t+j}))|E\right].\end{aligned}$$

Taking the ratio between two consecutive periods:

$$\frac{\frac{\partial s_{i,t+1}}{\partial w_{t+j+1}}}{\frac{\partial s_{i,t+1}}{\partial w_{t+j}}} = \beta\frac{S_{i,t+j+1}}{S_{i,t+j}}\frac{\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j+1})|E\right]}{\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j})|E\right]} \tag{24}$$

Notice that the Euler equation for an employed worker implies that

$$\beta\frac{\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j+1})|E\right]}{\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j})|E\right]} = \frac{1}{1+r} \tag{25}$$

Therefore,

$$\frac{\frac{\partial s_{i,t+1}}{\partial w_{t+j+1}}}{\frac{\partial s_{i,t+1}}{\partial w_{t+j}}} = \frac{S_{i,t+j+1}}{S_{i,t+j}}\frac{1}{1+r} \tag{26}$$

and

$$\frac{\partial s_{i,t+1}}{\partial w_{t+j+1}} = \frac{\partial s_{i,t+1}}{\partial w_{t+1}}\frac{S_{i,t+j+1}}{S_{i,t+1}}(1+r)^{-j} \tag{27}$$

Multiplying both sides by $h'(s_{t+1})$ and invoking the Chain Rule again yields the second equation in the Lemma. Q.E.D.

Proposition 1. Start from the decomposition in (10) in the text written in a slightly different form (the only difference is shifting the index j by one, so that it starts at zero):

$$\frac{\partial h_{i,1}}{\partial b_{j+1}} = \frac{\partial h_{i,1}}{\partial y_{j+1}} - \frac{\partial h_{i,1}}{\partial w_{j+1}}, \quad j \geq 0. \tag{28}$$

Substitute the results from Lemma 2 into this equation to obtain:

$$\frac{\partial h_{i,1}}{\partial b_{j+1}} = \frac{\partial h_{i,1}}{\partial y_1}(1+r)^{-j} - \frac{\partial h_{i,1}}{\partial w_1}\frac{S_{i,j+1}}{S_{i,1}}(1+r)^{-j}, \quad j \geq 0 \tag{29}$$

Sum this equation over $j = 0, \ldots, B_1 - 1$ to obtain $\frac{\partial h_{i,1}}{\partial \bar{b}_1}$:

$$\begin{aligned}
\frac{\partial h_{i,1}}{\partial \bar{b}_1} = \sum_{j=0}^{B_1-1} \frac{\partial h_{i,1}}{\partial b_{j+1}} &= \left(\sum_{j=0}^{B_1-1} \frac{1}{(1+r)^j}\right)\frac{\partial h_{i,1}}{\partial y_1} - \frac{1}{\tilde{S}_{i,1}}\left(\sum_{j=0}^{B_1-1}\frac{S_{i,j+1}}{(1+r)^{j+1}}\right)\frac{\partial h_{i,1}}{\partial w_1} \\
= \sum_{j=1}^{B_1} \frac{\partial h_{i,1}}{\partial b_j} &= \left(\sum_{j=1}^{B_1} \frac{1+r}{(1+r)^j}\right)\frac{\partial h_{i,1}}{\partial y_1} - \frac{1}{\tilde{S}_{i,1}}\left(\sum_{j=1}^{B_1}\frac{S_{i,j}}{(1+r)^{j}}\right)\frac{\partial h_{i,1}}{\partial w_1} \\
&\equiv \underbrace{(1+r)\tilde{B}_1(r)\frac{\partial h_{i,1}}{\partial y_1}}_{LIQ_{i,1}} - \underbrace{\frac{1}{\tilde{S}_{i,1}}\tilde{D}_{i,1}(r)\frac{\partial h_{i,1}}{\partial w_1}}_{MH_{i,1}},
\end{aligned} \tag{30}$$

where $\tilde{S}_{i,1} = (1+r)^{-1}S_{i,1}$, $\tilde{B}_1 = \sum_{t=1}^{B_1}(1+r)^{-t}$ and $\tilde{D}_{i,1} = \sum_{t=1}^{B_1}(1+r)^{-t}S_{i,t}$.

Sum also over $j = B_1, \ldots, B_1 + B_2 - 1$ to obtain $\frac{\partial h_{i,1}}{\partial \bar{b}_2}$:

$$\begin{aligned}
\frac{\partial h_{i,1}}{\partial \bar{b}_2} = \sum_{j=B_1}^{B_1+B_2-1} \frac{\partial h_{i,1}}{\partial b_{j+1}} &= \left(\sum_{j=B_1}^{B_1+B_2-1} \frac{1}{(1+r)^j}\right)\frac{\partial h_{i,1}}{\partial y_1} - \frac{1}{\tilde{S}_{i,1}}\left(\sum_{j=B_1}^{B_1+B_2-1}\frac{S_{i,j+1}}{(1+r)^{j+1}}\right)\frac{\partial h_{i,1}}{\partial w_1} \\
= \sum_{j=B_1+1}^{B_1+B_2} \frac{\partial h_{i,1}}{\partial b_j} &= \left(\sum_{j=B_1+1}^{B_1+B_2} \frac{1+r}{(1+r)^j}\right)\frac{\partial h_{i,1}}{\partial y_1} - \frac{1}{\tilde{S}_{i,1}}\left(\sum_{j=B_1+1}^{B_1+B_2}\frac{S_{i,j}}{(1+r)^{j}}\right)\frac{\partial h_{i,1}}{\partial w_1} \\
&\equiv \underbrace{(1+r)\tilde{B}_2(r)\frac{\partial h_{i,1}}{\partial y_1}}_{LIQ_{i,2}} - \underbrace{\frac{1}{\tilde{S}_{i,1}}\tilde{D}_{i,2}(r)\frac{\partial h_{i,1}}{\partial w_1}}_{MH_{i,2}},
\end{aligned} \tag{31}$$

where $\tilde{B}_2 = \sum_{t=B_1+1}^{B_1+B_2}(1+r)^{-t}$ and $\tilde{D}_{i,2} = \sum_{t=B_1+1}^{B_1+B_2}(1+r)^{-t}S_{i,t}$. Notice that, when $r = 0$, the terms simplify, so that $\tilde{B}_1(0) = B_1$, $\tilde{B}_2(0) = B_2$, $\tilde{D}_{i,1}(0) = D_{i,1}$, and $\tilde{D}_{i,2}(0) = D_{i,2}$.

The two equations (30) and (31) can be collected in matrix form as follows:

$$\begin{bmatrix} \frac{\partial h_{i,1}}{\partial \bar{b}_1} \\ \frac{\partial h_{i,1}}{\partial \bar{b}_2} \end{bmatrix} = \begin{bmatrix} (1+r)\tilde{B}_1(r) & -\frac{1}{S_{i,1}}\tilde{D}_{i,1} \\ (1+r)\tilde{B}_2(r) & -\frac{1}{S_{i,1}}\tilde{D}_{i,2} \end{bmatrix} \begin{bmatrix} \frac{\partial h_{i,1}}{\partial y_1} \\ \frac{\partial h_{i,1}}{\partial w_1} \end{bmatrix} \tag{32}$$

This matrix admits an inverse if $\frac{\tilde{D}_{i,1}(r)}{\tilde{B}_1(r)} \neq \frac{\tilde{D}_{i,2}(r)}{\tilde{B}_2}(r)$, which for $r = 0$ turns into $\frac{D_{i,1}}{B_1} \neq \frac{D_{i,2}}{B_2}$. Because $S_{i,t}$ is non-increasing in t, the condition is satisfied with $\frac{D_{i,1}}{B_1} > \frac{D_{i,2}}{B_2}$ if $h_{i,t} > 0$ at least once for $1 < t < B_1 + B_2$. Computing the inverse and pre-multiplying both sides of the equation with this inverse yields:

$$\begin{bmatrix} \frac{\partial h_{i,1}}{\partial y_1} \\ \frac{\partial h_{i,1}}{\partial w_1} \end{bmatrix} = \frac{1}{\tilde{B}_2(r)\frac{1+r}{\tilde{S}_{i,1}}\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\frac{1+r}{\tilde{S}_{i,1}}\tilde{D}_{i,2}(r)} \begin{bmatrix} -\frac{1}{\tilde{S}_{i,1}}\tilde{D}_{i,2}(r) & \frac{1}{\tilde{S}_{i,1}}\tilde{D}_{i,1}(r) \\ -(1+r)\tilde{B}_2(r) & (1+r)\tilde{B}_1(r) \end{bmatrix} \begin{bmatrix} \frac{\partial h_{i,1}}{\partial \bar{b}_1} \\ \frac{\partial h_{i,1}}{\partial \bar{b}_2} \end{bmatrix} \tag{33}$$

Therefore,

$$\frac{\partial h_{i,1}}{\partial y_1} = \frac{(1+r)^{-1}}{\tilde{B}_2(r)\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\tilde{D}_{i,2}(r)} \left(\tilde{D}_{i,1}(r)\frac{\partial h_{i,1}}{\partial \bar{b}_2} - \tilde{D}_{i,2}(r)\frac{\partial h_{i,1}}{\partial \bar{b}_1} \right)$$
$$\frac{\partial h_{i,1}}{\partial w_1} = \frac{\tilde{S}_{i,1}}{\tilde{B}_2(r)\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\tilde{D}_{i,2}(r)} \left(\tilde{B}_1(r)\frac{\partial h_{i,1}}{\partial \bar{b}_2} - \tilde{B}_2(r)\frac{\partial h_{i,1}}{\partial \bar{b}_1} \right). \tag{34}$$

Finally, substituting these results into (30) and (31) yields:

$$\begin{aligned}
LIQ_{i,1}(r) &= \left.\frac{\partial h_{i,1}}{\partial y}\right|_{B_1} = \frac{\tilde{B}_1(r)}{\tilde{B}_2(r)\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\tilde{D}_{i,2}(r)} \left(\tilde{D}_{i,1}(r)\frac{\partial h_{i,1}}{\partial \bar{b}_2} - \tilde{D}_{i,2}(r)\frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \\
MH_{i,1}(r) &= \left.\frac{\partial h_{i,1}}{\partial w}\right|_{B_1} = \frac{\tilde{D}_{i,1}(r)}{\tilde{B}_2(r)\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\tilde{D}_{i,2}(r)} \left(\tilde{B}_1(r)\frac{\partial h_{i,1}}{\partial \bar{b}_2} - \tilde{B}_2(r)\frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \\
LIQ_{i,2}(r) &= \left.\frac{\partial h_{i,1}}{\partial y}\right|_{B_2} = \frac{\tilde{B}_2(r)}{\tilde{B}_2(r)\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\tilde{D}_{i,2}(r)} \left(\tilde{D}_{i,1}(r)\frac{\partial h_{i,1}}{\partial \bar{b}_2} - \tilde{D}_{i,2}(r)\frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \\
MH_{i,2}(r) &= \left.\frac{\partial h_{i,1}}{\partial w}\right|_{B_2} = \frac{\tilde{D}_{i,2}(r)}{\tilde{B}_2(r)\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\tilde{D}_{i,2}(r)} \left(\tilde{B}_1(r)\frac{\partial h_{i,1}}{\partial \bar{b}_2} - \tilde{B}_2(r)\frac{\partial h_{i,1}}{\partial \bar{b}_1} \right)
\end{aligned} \tag{35}$$

Setting $r = 0$ leads to the expressions in the Proposition. Q.E.D.

Proposition 2. The planner solves the following problem:

$$V^P(\mathbf{b}, \tau) = \max \int V_{i,0}(\omega_{i,0})\, di + \lambda \left(G(\mathbf{b}, \tau) - \bar{G} \right), \tag{36}$$

where

$$\begin{aligned}
G(\mathbf{b}, \tau) &= \tau \sum_{t=1}^{T} (1+r)^{-t}(1 - S_t) - \bar{b}_1 \sum_{t=1}^{B_1} (1+r)^{-t} S_t - \bar{b}_2 \sum_{t=B_1+1}^{B_1+B_2} (1+r)^{-t} S_t \\
&= \tau \sum_{t=1}^{T} (1+r)^{-t} - \tau \tilde{D}(r) - \bar{b}_1 \tilde{D}_1(r) - \bar{b}_2 \tilde{D}_2(r),
\end{aligned} \tag{37}$$

where we have used $\tilde{D}_1(r) = \sum_{t=1}^{B_1}(1+r)^{-t}S_t$, $\tilde{D}_2(r) = \sum_{t=B_1+1}^{B_1+B_2}(1+r)^{-t}S_t$, and $\tilde{D}(r) = \sum_{t=1}^{T}(1+r)^{-t}$.

For any agent i:

$$\begin{aligned}
\frac{\partial}{\partial b_t} V_{i,0}(\omega_{i,0}) &= \frac{\partial}{\partial b_t} \mathbb{E}_0 V_{i,1}(\omega_{i,1}) \\
&= -\frac{1}{\Lambda(\omega_{i,0})} \frac{\partial s_{i,1}(\omega_{i,0})}{\partial b_t} \\
&= -\frac{1}{h'_{i,1}(s_{i,1}(\omega_{i,0}))\Lambda(\omega_{i,0})} \frac{\partial h_{i,1}(s_{i,1}(\omega_{i,0}))}{\partial b_t}
\end{aligned} \tag{38}$$

Assuming that agents are ex-ante homogeneous, so that $\forall i : h_{i,1}(s) = h_1(s)$ and $\forall i : \omega_{i,0} = \omega_0$,

$$\begin{aligned}\frac{\partial}{\partial b_t}\int V_{i,0}(\omega_{i,0})\,di &= \int \frac{\partial}{\partial b_t} V_{i,0}(\omega_{i,0})\,di \\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\int \frac{\partial h_{i,1}(s_{i,1}(\omega_{i,0}))}{\partial b_t}\,di \\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\frac{\partial h_1(s_1(\omega_0))}{\partial b_t} \end{aligned} \tag{39}$$

and

$$\begin{aligned}\frac{\partial}{\partial \bar{b}_1}\int V_{i,0}(\omega_{i,0})\,di &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\sum_{t=1}^{B_1}\frac{\partial h_1(s_1(\omega_0))}{\partial b_t} \\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}[LIQ_1(r) - MH_1(r)] \end{aligned} \tag{40}$$

Equivalently,

$$\begin{aligned}\frac{\partial}{\partial \bar{b}_2}\int V_{i,0}(\omega_{i,0})\,di &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\sum_{t=B_1+1}^{B_1+B_2}\frac{\partial h_1(s_1(\omega_0))}{\partial b_t} \\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}[LIQ_2(r) - MH_2(r)] \end{aligned} \tag{41}$$

Consider first the impact of changing τ in only one period, i.e., a movement in τ_t:

$$\begin{aligned}\frac{\partial}{\partial \tau_t}V_{i,0}(\omega_{i,0}) &= -\frac{\partial}{\partial w_t}V_{i,0}(\omega_{i,0}) \\ &= -\beta^t(1 - S_{i,t})\mathbb{E}_0^e\left[v_{i,1}^e(c_{i,t}^e)\right] \\ &= -\frac{1 - S_{i,t}}{S_{i,t}}\frac{\partial}{\partial w_t}[\mathbb{E}_0 V_{i,1}^e(\omega_{i,1}) - \mathbb{E}_0 V_{i,1}^u(\omega_{i,1})] \\ &= -\frac{1 - S_{i,t}}{S_{i,t}}\frac{1}{\Lambda(\omega_{i,0})}\frac{\partial s_{i,1}(\omega_{i,0})}{\partial w_t} \\ &= -\frac{1 - S_{i,t}}{S_{i,t}}\frac{1}{h'_{i,1}(s_{i,1}(\omega_{i,0}))\Lambda(\omega_{i,0})}\frac{\partial h_{i,1}(s_{i,1}(\omega_{i,0}))}{\partial w_t} \end{aligned} \tag{42}$$

Using ex-ante homogeneity

$$\begin{aligned}\frac{\partial}{\partial \tau_t}\int V_{i,0}(\omega_{i,0})\,di &= \int \frac{\partial}{\partial \tau_t}V_{i,0}(\omega_{i,0})\,di \\ &= -\frac{1 - S_t}{S_t}\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\int \frac{\partial h_{i,1}(s_{i,1}(\omega_{i,0}))}{\partial w_t}\,di \\ &= -\frac{1 - S_t}{S_t}\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\frac{\partial h_1(s_1(\omega_0))}{\partial w_t} \end{aligned} \tag{43}$$

Therefore,

$$
\begin{aligned}
\frac{\partial}{\partial \tau} \int V_{i,0}(\omega_{i,0})\, di &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)} \sum_{t=1}^{T} \frac{1-S_t}{S_t} \frac{\partial h_1(s_1(\omega_0))}{\partial w_t} \\
&= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)} \sum_{t=1}^{T} \frac{1-S_t}{S_t} \frac{S_t}{S_1(1+r)^{t-1}} \frac{\partial h_1(s_1(\omega_0))}{\partial w_1} \\
&= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)} \frac{\partial h_1(s_1(\omega_0))}{\partial w_1} \sum_{t=1}^{T} \frac{1-S_t}{S_1(1+r)^{t-1}} \\
&= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)} \frac{\partial h_1(s_1(\omega_0))}{\partial w_1} \frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{S}_1(r)}, \qquad (44)
\end{aligned}
$$

where $\tilde{T}(r) = \sum_{t=1}^{T}(1+r)^{-t}$ and $\tilde{D}(r)$ and $\tilde{S}_1(r)$ are defined as in the proof of Proposition 1. Notice that, from the proof of Proposition 1,

$$
\begin{aligned}
MH_1(r) &= \frac{1}{\tilde{S}_1} \tilde{D}_1(r) \frac{\partial h_1(s_1(\omega_0))}{\partial w_1} \\
MH_2(r) &= \frac{1}{\tilde{S}_1} \tilde{D}_2(r) \frac{\partial h_1(s_1(\omega_0))}{\partial w_1} \qquad (45)
\end{aligned}
$$

Using these relationships, the derivative with respect to τ can be written in terms of MH_1 and MH_2:

$$
\begin{aligned}
\frac{\partial}{\partial \tau} \int V_{i,0}(\omega_{i,0})\, di &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)} \frac{\partial h_1(s_1(\omega_0))}{\partial w_1} \frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{S}_1(r)} \\
&= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)} \frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_1(r)} MH_1(r) \qquad (46) \\
&= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)} \frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_2(r)} MH_2(r) \qquad (47)
\end{aligned}
$$

At an interior optimum

$$
\begin{aligned}
\frac{\partial}{\partial \bar{b}_1} \int V_{i,0}(\omega_{i,0})\, di &= -\lambda \frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_1} \\
-\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)} [LIQ_1(r) - MH_1(r)] &= -\lambda \frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_1} \qquad (48)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial}{\partial \tau} \int V_{i,0}(\omega_{i,0})\, di &= -\lambda \frac{\partial G(\mathbf{b},\tau)}{\partial \tau} \\
-\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)} \frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_1(r)} MH_1(r) &= -\lambda \frac{\partial G(\mathbf{b},\tau)}{\partial \tau} \qquad (49)
\end{aligned}
$$

Taking the ratio:

$$
\begin{aligned}
-\frac{LIQ_1(r) - MH_1(r)}{\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_1(r)} MH_1(r)} &= -\frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_1} / \frac{\partial G(\mathbf{b},\tau)}{\partial \tau} \\
-\frac{LIQ_1(r) - MH_1(r)}{MH_1(r)} &= -\frac{\tilde{T}(r) - \tilde{D}(r)}{\tilde{D}_1(r)} \frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_1} / \frac{\partial G(\mathbf{b},\tau)}{\partial \tau} \\
-\frac{LIQ_1(r)}{MH_1(r)} &= -\frac{\tilde{T}(r) - \tilde{D}(r)}{\tilde{D}_1(r)} \frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_1} / \frac{\partial G(\mathbf{b},\tau)}{\partial \tau} - 1 \qquad (50)
\end{aligned}
$$

Following similar steps,

$$
\begin{aligned}
-\frac{LIQ_2(r) - MH_2(r)}{\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_2(r)} MH_2(r)} &= -\frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_2} / \frac{\partial G(\mathbf{b},\tau)}{\partial \tau} \\
-\frac{LIQ_2(r) - MH_2(r)}{MH_2(r)} &= -\frac{\tilde{T}(r) - \tilde{D}(r)}{\tilde{D}_2(r)} \frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_2} / \frac{\partial G(\mathbf{b},\tau)}{\partial \tau} \\
-\frac{LIQ_2(r)}{MH_2(r)} &= -\frac{\tilde{T}(r) - \tilde{D}(r)}{\tilde{D}_2(r)} \frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_2} / \frac{\partial G(\mathbf{b},\tau)}{\partial \tau} - 1 \qquad (51)
\end{aligned}
$$

Notice that, by the Implicit Function Theorem,

$$
-\frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_k} / \frac{\partial G(\mathbf{b},\tau)}{\partial \tau} = \frac{\partial \tau}{\partial \bar{b}_k}|_{G(\mathbf{b},\tau)=\bar{G}} \qquad (52)
$$

$$
G(\mathbf{b},\tau) = \bar{G} \Leftrightarrow \tau(\tilde{T}(r) - \tilde{D}(r)) = \overline{G} + \bar{b}_1 \tilde{D}_1(r) + \bar{b}_2 \tilde{D}_2(r) \qquad (53)
$$

Therefore,

$$
\frac{\partial \tau}{\partial \bar{b}_1}(\tilde{T}(r) - \tilde{D}(r)) - \tau \frac{\partial \tilde{D}(r)}{\partial \bar{b}_1} = \tilde{D}_1(r) + \bar{b}_1 \frac{\partial \tilde{D}_1(r)}{\partial \bar{b}_1} + \bar{b}_2 \frac{\partial \tilde{D}_2(r)}{\partial \bar{b}_1} \qquad (54)
$$

and

$$
\begin{aligned}
\frac{\partial \tau}{\partial \bar{b}_1} &= \frac{1}{\tilde{T}(r) - \tilde{D}(r)} \left[\tilde{D}_1(r) + \bar{b}_1 \frac{\partial \tilde{D}_1(r)}{\partial \bar{b}_1} + \bar{b}_2 \frac{\partial \tilde{D}_2(r)}{\partial \bar{b}_1} + \tau \frac{\partial \tilde{D}(r)}{\partial \bar{b}_1} \right] \\
&= \frac{\tilde{D}_1(r)}{\tilde{T}(r) - \tilde{D}(r)} \left[1 + \varepsilon_{\tilde{D}_1,\bar{b}_1} + \frac{\bar{b}_2 \tilde{D}_2(r)}{\bar{b}_1 \tilde{D}_1(r)} \varepsilon_{\tilde{D}_2,\bar{b}_1} + \frac{\tau \tilde{D}(r)}{\bar{b}_1 \tilde{D}_1(r)} \varepsilon_{\tilde{D},\bar{b}_1} \right] \qquad (55)
\end{aligned}
$$

Substituting into (50):

$$
\begin{aligned}
-\frac{LIQ_1(r)}{MH_1(r)} &= \frac{\tilde{T}(r) - \tilde{D}(r)}{\tilde{D}_1(r)} \left(-\frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_1} / \frac{\partial G(\mathbf{b},\tau)}{\partial \tau} \right) - 1 \\
&= \frac{\tilde{T}(r) - \tilde{D}(r)}{\tilde{D}_1(r)} \frac{\partial \tau}{\partial \bar{b}_1}|_{G(\mathbf{b},\tau)=\bar{G}} - 1 \\
&= \varepsilon_{\tilde{D}_1,\bar{b}_1} + \frac{\bar{b}_2 \tilde{D}_2(r)}{\bar{b}_1 \tilde{D}_1(r)} \varepsilon_{\tilde{D}_2,\bar{b}_1} + \frac{\tau \tilde{D}(r)}{\bar{b}_1 \tilde{D}_1(r)} \varepsilon_{\tilde{D},\bar{b}_1} \qquad (56)
\end{aligned}
$$

Setting $r = 0$ yields the first expression in the proposition.

Analogously,

$$\frac{\partial \tau}{\partial \bar{b}_2} = \frac{1}{\tilde{T}(r) - \tilde{D}(r)} \left[\tilde{D}_2(r) + \bar{b}_2 \frac{\partial \tilde{D}_2(r)}{\partial \bar{b}_1} + \bar{b}_1 \frac{\partial \tilde{D}_1(r)}{\partial \bar{b}_2} + \tau \frac{\partial \tilde{D}(r)}{\partial \bar{b}_2} \right]$$
$$= \frac{\tilde{D}_2(r)}{\tilde{T}(r) - \tilde{D}(r)} \left[1 + \varepsilon_{\tilde{D}_2, \bar{b}_2} + \frac{\bar{b}_1 \tilde{D}_1(r)}{\bar{b}_2 \tilde{D}_2(r)} \varepsilon_{\tilde{D}_1, \bar{b}_2} + \frac{\tau \tilde{D}(r)}{\bar{b}_2 \tilde{D}_2(r)} \varepsilon_{\tilde{D}, \bar{b}_2} \right] \tag{57}$$

and

$$-\frac{LIQ_2(r)}{MH_2(r)} = \frac{\tilde{T}(r) - \tilde{D}(r)}{\tilde{D}_2(r)} \left(-\frac{\partial G(\mathbf{b}, \tau)}{\partial \bar{b}_2} / \frac{\partial G(\mathbf{b}, \tau)}{\partial \tau} \right) - 1$$
$$= \frac{\tilde{T}(r) - \tilde{D}(r)}{\tilde{D}_2(r)} \frac{\partial \tau}{\partial \bar{b}_2} |_{G(\mathbf{b}, \tau) = \bar{G}} - 1$$
$$= \varepsilon_{\tilde{D}_2, \bar{b}_2} + \frac{\bar{b}_1 \tilde{D}_1(r)}{\bar{b}_2 \tilde{D}_2(r)} \varepsilon_{\tilde{D}_1, \bar{b}_2} + \frac{\tau \tilde{D}(r)}{\bar{b}_2 \tilde{D}_2(r)} \varepsilon_{\tilde{D}, \bar{b}_2} \tag{58}$$

Setting $r = 0$ yields the second expression in the proposition. Q.E.D.

Proposition 3. The quadratic specification implies that $\Lambda(\omega_{i,0}) = \frac{1}{\psi} > 0$ does not depend on the state of the world. In addition, the linear deterministic relationship between search effort and $h_{i,t}$ implies that $h'_{i,1}(s(\omega_{i,0})) = 1$ does also not depend in the state of the world. Therefore, $\frac{1}{h'_{i,1}(s(\omega_{i,0}))\Lambda(\omega_{i,0})} = \psi$ can be taken out of the expectations over i without assuming that agents share the same $\omega_{i,0}$. Thus, the exact same steps as in the proof of Proposition 2 can be retraced leading to the same result in the end. Q.E.D.

A.2 Extension: additional moments for the moral hazard effect

Additional moment

The difference in entitlements in the population and the clean thresholds at which they occur allow for the use of additional moment conditions to identify the moral hazard effect. For this, we take advantage of the theoretical results by Landais (2015), who shows that the response of hazard rates to changes in the entitlement period can be useful in identifying the moral hazard effect. His result adapted to our setting and notation relates the moral-hazard effect to the following linear combination of derivatives:

$$-MH_2 \left(1 - \frac{B_2 S_{B_1 + B_2}}{D_{i,2}} \right) = \frac{\partial h_1}{\partial \bar{b}_2} - \frac{B_2}{\bar{b}_2} \frac{\partial h_1}{\partial B_2}, \tag{59}$$

where $S_{B_2+B_2}$ is the survival rate at the time when employment benefits expire and $\frac{\partial h_1}{\partial B_2}$ is the change in the first-period hazard rate induced by a change in the length of the entitlement to unemployment benefits.[1]

[1] A subtle point when this equation is taken to the data is that benefits $\bar{b}_2$ have to be expressed in terms of the same time unit as B_2.

The expression in (59) is obtained as follows. First, adapting the key insight behind Proposition 1 by Landais (2015) to our environment, we obtain:

$$\frac{\partial h_{i,1}}{\partial B_2} \approx \bar{b}_2 \frac{\partial h_{i,1}}{\partial b_{B_1+B_2+\Delta}} = \bar{b}_2 \left(\frac{\partial h_{i,1}}{\partial y_1} - \frac{S_{i,B_1+B_2+\Delta}}{S_{i,1}} \frac{\partial h_{i,1}}{\partial w_1} \right), \tag{60}$$

where $\Delta \geq 0$ is the length of a time-step. The approximation will be better for smaller time-steps. The last equality follows directly from (29). Solving this equation for $\frac{\partial h_{i,1}}{\partial y_1}$ yields

$$\frac{\partial h_{i,1}}{\partial y_1} \approx \frac{1}{\bar{b}_2} \frac{\partial h_{i,1}}{\partial B_2} + \frac{S_{i,B_1+B_2+\Delta}}{S_{i,1}} \frac{\partial h_{i,1}}{\partial w_1} \tag{61}$$

The decomposition of a change in $\bar{b}_2$ into a liquidity and moral hazard effect is given by (31):

$$\frac{\partial h_{i,1}}{\partial \bar{b}_2} = \underbrace{B_2 \frac{\partial h_{i,1}}{\partial y_1}}_{LIQ_{i,2}} - \underbrace{\frac{1}{S_{i,1}} D_{i,2} \frac{\partial h_{i,1}}{\partial w_1}}_{MH_{i,2}}, \tag{62}$$

Substituting $\frac{\partial h_{i,1}}{\partial y_1}$ into this equation,

$$\frac{\partial h_{i,1}}{\partial \bar{b}_2} \approx B_2 \left(\frac{1}{\bar{b}_2} \frac{\partial h_{i,1}}{\partial B_2} + \frac{S_{i,B_1+B_2+\Delta}}{S_{i,1}} \frac{\partial h_{i,1}}{\partial w_1} \right) - MH_{i,2}. \tag{63}$$

After rearranging and expressing $\frac{\partial h_{i,1}}{\partial w_1}$ in terms of $MH_{i,2}$, we obtain:

$$-MH_{i,2} \left(1 - \frac{B_2 S_{i,B_1+B_2+\Delta}}{D_{i,2}} \right) \approx \frac{\partial h_{i,1}}{\partial \bar{b}_2} - \frac{B_2}{\bar{b}_2} \frac{\partial h_{i,1}}{\partial B_2} \tag{64}$$

and setting $\Delta = 0$:

$$-MH_{i,2} \left(1 - \frac{B_2 S_{i,B_1+B_2}}{D_{i,2}} \right) \approx \frac{\partial h_{i,1}}{\partial \bar{b}_2} - \frac{B_2}{\bar{b}_2} \frac{\partial h_{i,1}}{\partial B_2}. \tag{65}$$

GMM

The moral hazard effect can be eliminated using the last equation in (15) in the main text.

$$-\frac{D_{i,2}}{B_2 D_{i,1} - B_1 D_{i,2}} \left(B_1 \frac{\partial h_{i,1}}{\partial \bar{b}_2} - B_2 \frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \left(1 - \frac{B_2 S_{i,B_1+B_2}}{D_{i,2}} \right) = \frac{\partial h_{i,1}}{\partial \bar{b}_2} - \frac{B_2}{\bar{b}_2} \frac{\partial h_{i,1}}{\partial B_2} \tag{66}$$

Solving this equation for $\frac{\partial h_{i,1}}{\partial B_2}$:

$$\frac{\partial h_{i,1}}{\partial B_2} = -\bar{b}_2 \Xi \frac{\partial h_{i,1}}{\partial \bar{b}_1} + \frac{\bar{b}_2}{B_2} (\Xi B_1 - 1) \frac{\partial h_{i,1}}{\partial \bar{b}_2}, \tag{67}$$

where $\Xi \equiv \left(1 - \frac{B_2 S_{i,B_1+B_2}}{D_{i,2}} \right) \frac{D_{i,2}}{B_2 D_{i,1} - B_1 D_{i,2}}$.

Using this theoretical relationship, we define two error terms in terms of the variables of interest:

$$\epsilon_{i,1}(\theta_1,\theta_2) \equiv y_i - \left(\alpha + x_i'\eta + \gamma(v_i - k_1) - \sum_{j=1}^{2}\frac{\theta_j}{r_j}W_{i,j}(v_i - k_j)\right),$$

$$\epsilon_{i,2}(\theta_1,\theta_2) \equiv y_i - \left(\tilde{\alpha} + x_i'\tilde{\eta} + \tilde{\gamma}(d_i - \bar{d}) - \bar{b}_2\left(\Xi\theta_1 - \frac{\Xi B_1 - 1}{B_2}\theta_2\right)\tilde{W}_i(d_i - \bar{d})\right). \tag{68}$$

The first error expresses the RKD specification in terms of the variables of interest, θ_1 and θ_2. The second error is a RDD specification with running variable d_i (days in prior jobs eligible for unemployment benefits in the current spell) and threshold $\bar{d}$, the number of days at which observation i switches from one entitlement period to the next. For ease of notation, we have expressed these equations using only first-degree polynomials. They can be generalized to higher-degree polynomials by adding the appropriate terms. Each of these equations has its own bandwidth parameter, which governs which observations are included in the estimation. Using these error terms in the moment contributions, the parameters θ_1 and θ_2 can be estimated via GMM using standard methods.

Results

In the case of Spain, entitlements are determined by thresholds in the number of days worked before the unemployment spell. The derivative $\frac{\partial h_1}{\partial B_2}$ can therefore be estimated using a regression discontinuity design (RDD) using the number of days worked as the running variable. Because of our prior result that liquidity effects are relatively small, we expect a small coefficient for this derivative, which is directly related to the liquidity effect.

In the first column of Table 1 we show the estimate of this derivative using all entitlement thresholds in our estimation sample simultaneously. The point estimate is of the expected sign and small. It is not significantly different from zero. We experimented with estimating this coefficient separately for each threshold and for various bandwidth choices and found small and insignificant estimates in all cases.

At first glance, this result seems to corroborate our finding that moral hazard effects dominate over liquidity effects. When we combine this new moment with the moments estimated in our main results, we find evidence for even smaller liquidity effects, which are completely eclipsed by moral hazard effects. To combine the various moment conditions, we estimate a system that imposes the cross-equation restrictions implied by the theory using a GMM estimator. The results are shown in the second column of Table 1.

The GMM estimation yields a stronger estimated effect for $\bar{b}_1$ and a weaker effect for $\bar{b}_2$. The effect of $\overline{B}_2$ is omitted because we substituted it to obtain the cross-equation restrictions. Plugging these estimates into the equation that solves for the moral hazard and liquidity effect yields an even larger moral hazard effect than the one obtained in our main results. In fact, these estimated coefficients would imply the second period moral hazard effect is 100% of the total, and would lead to the conclusion that unemployment benefits in the second period are too high, regardless of the magnitude estimated for the fiscal cost (as long as it is strictly positive). This statement does not imply that *any* positive level of unemployment benefits is too high and

Table 1: Estimates using an additional moment

	(1) RDD	(2) GMM
$\frac{\partial h_1}{\partial \bar{b}_1}$		-0.018***
		(0.005)
$\frac{\partial h_1}{\partial \bar{b}_2}$		-0.010***
		(0.003)
$\frac{\partial h_1}{\partial B_2}$	-0.003	
	(0.068)	
Observations	6,954	64,987

Note: Estimations include controls for year and month dummies, age (at the time of becoming unemployed) and age squared, a dummy variable for being male, a dummy for having a permanent contract in the previous job, dummies for the qualifications of the job, for the number of the unemployment spell, and dummies for regions.

that they should therefore be set to zero. As discussed in the statistical extrapolation exercise, the estimates have a local nature and do not impose restrictions on the ratio of liquidity to moral hazard effects for benefit levels that are distant from those observed in practice. A valid takeaway from this section is that taking into account moment conditions based on the length of unemployment benefit coverage appears to reinforce the conclusion that unemployment benefits are too high in the second period of the unemployment spell.

B Robustness checks

B.1 Sensitivity to bandwidth choice, polynomial order, and covariates

Our estimates are robust to the use of different bandwidths. As noted by other authors, for instance Landais (2015), a regression kink design is more demanding in terms of bandwidth size than a regression discontinuity design. In Figure 1 we plot the point estimates for the probability of exiting unemployment in the first period for different bandwidths along with 95% confidence intervals. We find that main results vary very little with the bandwidth choice, with less precise estimates at the second kink for small bandwidth sizes. In particular, the relative importance of liquidity and moral hazard effects remains remarkably constant. Observations in the region between kinks are always included, except if they fall outside the bandwidth of both kinks.

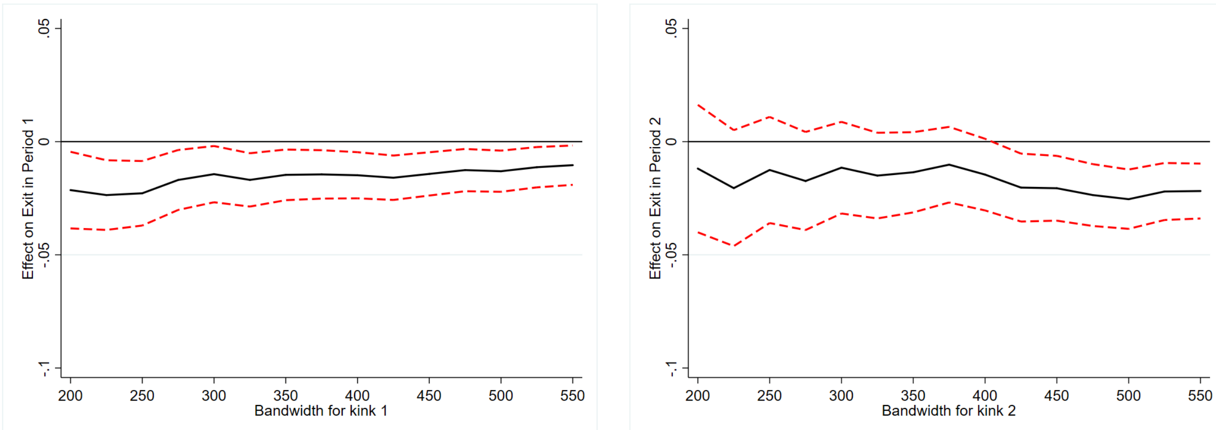

Figure 1: Estimates on the probability of exiting unemployment in the first period for different bandwidths, with 95% confidence intervals.

Regarding the polynomial order choice, even though point estimates change, moral hazard continues to dominate in both periods. More importantly, in both cases benefits of unemployment insurance are low relative to the costs. The Akaike Criterion, whose results are presented in the last column of Table 2, selects the quadratic specification as the preferred specification, although all values are very similar.

Table 2: Summary of results using linear or quadratic polynomials.

	(1) MH Period 1	(2) Optimal Period 1	(3) MH Period 2	(4) Optimal Period 2	(5) AIC
Linear	79%	Too high	64%	Too high	84377
Quadratic	86%	Too high	74%	Too high	84370
Cubic	85%	Too high	72%	Too high	84372

Note: The table summarizes results from using a linear, a quadratic, and a cubic specification. The value for MH represents the relative importance of the moral hazard effect, Optimal denotes if unemployment benefits are too high or too low with respect to optimal levels, and AIC denotes Akaike's Information Criterion.

Finally, in Table 3 we present the results from estimating our baseline equation when no covariates are included. Results remain practically identical.

In order to detect whether the inclusion of the long-term unemployed is affecting the results, we also repeated the regression in the first column of Table 3 on a reduced sample, excluding observations in which the unemployment spell lasts for the full period of coverage or longer. The expected sign of the effect of this sample restriction is ex-ante ambiguous because a change in the sample will also lead to a shift in the polynomial that approximates the smooth relationship

Table 3: RKD estimations on several outcomes: Period 1992 - 2012, workers between 30 and 50 years old

	(1) Exit in period 1	(2) Duration period 1	(3) Duration period 2	(4) Non-employment duration	(5) MH	(6) Optimal
θ_1	-0.018***	0.019**	0.123***	0.154***	82%	Too high
	(0.006)	(0.008)	(0.025)	(0.031)		
θ_2	-0.027***	0.029**	0.178***	0.213***	68%	Too high
	(0.009)	(0.011)	(0.037)	(0.047)		
Observations	61,971	61,971	61,971	61,971		

Note: All estimates are from specifications with no covariates, for the quadratic case. Duration in each period is measured as days in unemployment in each period. Total duration is days in non-employment. Coefficients are transformed in order to obtain the values of interest: the impact of increasing benefits in each period on each outcome. The value for MH represents the relative importance of the moral hazard effect, and Optimal shows if unemployment benefits are too high or too low with respect to optimal levels.

between the running variable and the dependent variable. The magnitudes of the point estimates are lower than in the baseline at the first kink and similar at the second, although the precision in the estimates does not lead to a rejection of the hypothesis that they are equal to the baseline at the usual confidence levels.

B.2 Placebo kinks and permutation tests

Following Landais (2015) and Kolsrud et al. (2018) we add a test aimed at detecting the kinks assuming that the actual location of the kinks is not known. We estimate our baseline equation for a range of placebo kinks and compare the R-squared obtained in each estimation. The placebo kinks are placed in EUR 25 increments from the true location of the kinks (we move both kinks outward or inward at the same time). We cannot use a wide range for the placebo kinks because both actual kinks are relatively close. The location in which the R-squared is maximized is situated at a EUR 25 difference from the real kink points. We present in Figure 2 the evolution of the R-squared for different locations of the kinks. We observe that R-squared is similar in a range of EUR 25, and that it drops when we move farther away.

We also use placebo kinks in a different way, based on the permutation procedure suggested by Ganong and Jaeger (2018) . This strategy implies testing the significance of our parameters of interest using a range of placebo kinks instead of the actual location of each kink. The main argument, adapted to our problem, is that if the true relationship between the probability of leaving unemployment and pre-unemployment earnings is highly non-linear, many placebo kinks will show significant and large estimates.

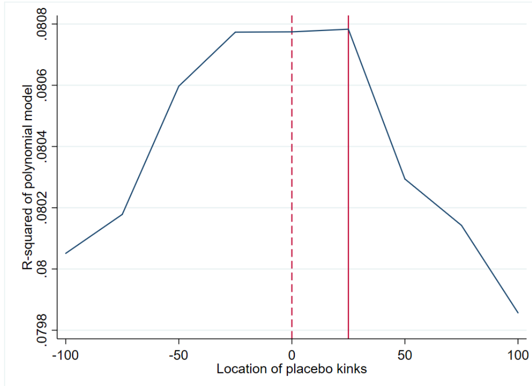

Figure 2: R-squared for different locations of the kinks.

Note: We plot the R-squared corresponding to the baseline regression computed using placebo kinks, represented in euros relative to the actual kinks, set at 0. The solid line shows the kink value at which the R-squared is maximized.

The permutation procedure by Ganong and Jaeger (2018) assesses whether the true coefficient estimate is larger than those at placebo kinks placed away from the true kink. This procedure allows to compute 95% confidence intervals for the parameters of interest.[2] We transform these confidence intervals and calculate the corresponding standard errors and show them in Table 4. In general, standard errors are similar to the robust standard errors in our baseline estimation.

[2]We base our computations on the Stata codes made available by Kolsrud et al. (2018).

Table 4: RKD estimations on several outcomes: Period 2005 - 2012, workers between 30 and 50 years old

	(1) Exit in period 1	(2) Duration period 1	(3) Duration period 2	(4) Non-employment duration
θ_1	-0.014**	0.014*	0.095***	0.120***
Robust s.e.	(0.006)	(0.007)	(0.024)	(0.030)
Perm. Test s.e.	(0.008)	(0.001)	(0.035)	(0.042)
θ_2	-0.021**	0.022*	0.142***	0.168**
Robust s.e.	(0.009)	(0.011)	(0.036)	(0.045)
Perm. Test s.e.	(0.010)	(0.003)	(0.057)	(0.060)

Note: We present the estimates from our baseline equation. We include robust standard errors from the baseline estimation and standard errors from the permutation test method by Ganong and Jaeger (2018). Coefficients are transformed in order to obtain the values of interest: the impact of increasing benefits in each period on each outcome.

C Monte Carlo exercise

In the empirical application in this paper, the unemployment benefit scheme exhibits two kinks in the relationship between pre-unemployment earnings and unemployment benefits. Most of the econometric tools developed in the RKD literature are designed for the case of only one kink.

We compare results from two alternative estimation strategies in the presence of two kinks using a Monte Carlo procedure. In a first strategy, we treat the kinks as independent, and perform an estimation separately at each kink disregarding the existence of the other. This strategy follows the procedure in the classical situation with one kink. In the second strategy—the one that we used in this paper—we estimate a single equation including both kinks simultaneously. We compare both strategies to evaluate their performance in a setting that emulates the main characteristics of the case analyzed in this paper and find that they both work equally well.

C.1 Setup

Our Monte Carlo procedure takes into account the characteristics of the unemployment insurance scheme in Spain in the period 1992-2012. The level of benefits is set at $r_1 = 70\%$ of prior labor income during the first six months in unemployment, and at $r_2 = 60\%$ during the remainder of the period in which the worker is entitled to unemployment benefits.

The data for the Monte Carlo simulation are generated using the following equation, which is linear in V:[3]

$$Y_i = 1 + 0.30V_i + 0.10b_{1i} + 0.15b_{2i} + u_i \quad i = 1, ..., N,$$

where u_i is sampled from a Normal $(0, 0.25)$ and $V_i = \exp(z_i)500 + 1000$, whit z_i sampled from a Normal $(0, 1)$. We consider a sample size of $N \in \{1,000; 2,000; 5,000\}$ in each simulation, and conduct $5,000$ replications. We use three different bandwidths $h \in \{200, 350, 500\}$.

First strategy: one equation per kink

In the first strategy we estimate one equation for each kink separately. Therefore, the estimates for θ_1 and θ_2 are obtained independently from each other from the following equations:

$$E[Y|V = v] = \alpha + \gamma_1(v - k_1) + \beta_{j1}W_j(v - k_j), \quad j = 1, 2, \tag{69}$$

where $W_j = 1$ for those observations above the corresponding kink. The equation is estimated for a bandwidth h, using observations such that $|V - k_j| < h$. Then, we compute $\hat{\theta}_j = -\hat{\beta}_{j1}/r_j$, $j = 1, 2$. This is the usual strategy if only one kink is present.

[3]We construct V using a distribution similar to that observed in our dataset. We set the values of the parameters close to the point estimates obtained in our empirical exercise for total duration. Conclusions about the relative performance of the strategies are unaffected by changes in these values.

Second strategy: two kinks in the same equation

In the second strategy, we estimate a single equation to obtain the two parameters of interest, as in the paper. The equation we estimate is:

$$E[Y|V = v] = \alpha + \gamma_1(v - k_1) + \sum_{j=1}^{2} \beta_{j1} W_j(v - k_j), \tag{70}$$

where W_j is equal to 1 if pre-unemployment earnings are above kink j ($v \geq k_j, j = 1, 2$).

C.2 Results

We show the main results of the analysis in Table 5 for θ_1 and in Table 6 for θ_2. We present the mean values, the standard deviation, and 95% confidence intervals for the corresponding estimates for $\hat{\theta}_j = -\hat{\beta}_j/r_j$, $j = 1, 2$ from each strategy for $N = 5,000$.

Table 5: Monte Carlo results for θ_1 (true value: $\theta_1 = 0.10$)

Strategy	h	Mean	Std. Dev.	95% Conf. Interval	
Two equations	200	.0999957	.0013465	.0977691	.1022019
Two equations	350	.1000027	.0005675	.0990646	.100933
Two equations	500	.099996	.0003993	.0993362	.1006439
One equation	200	.1000077	.0008388	.0986043	.1013617
One equation	350	.1000055	.0004742	.0992114	.1007861
One equation	500	.1000005	.000349	.0994151	.1005778

Note: Results from Monte Carlo simulation using 5,000 replications and 5,000 observations in each replication. We use three different bandwidths in each strategy.

Table 6: Monte Carlo results for θ_2 (true value: $\theta_2 = 0.15$)

Strategy	h	Mean	Std. Dev.	95% Conf. Interval	
Two equations	200	.1500265	.0020791	.146672	.1534579
Two equations	350	.1499993	.000895	.1485171	.1514795
Two equations	500	.1499934	.0006254	.1489811	.1510052
One equation	200	.1500283	.001264	.1479446	.1521159
One equation	350	.1500122	.0007248	.1488294	.1511856
One equation	500	.1499981	.0005347	.1491137	.1508795

Note: Results from Monte Carlo simulation using 5,000 replications and 5,000 observations in each replication. We use three different bandwidths in each strategy.

To complete the analysis, in Tables 7 and 8 we show the proportion of rejections of the null hypothesis $H_0 : \theta_1 = 0.10$ versus $H1 : \theta_1 \neq 0.10$ and $H_0 : \theta_2 = 0.15$ versus $H1 : \theta_2 \neq 0.15$. As expected, precision increases with sample size, but results are good even with small sample sizes. We observe that both null hypotheses are rejected in approximately the same proportion as the significance level (5%) in almost all cases.

Table 7: Monte Carlo results for the proportion of rejections of $H_0 : \theta_1 = 0.10$

	$N = 2,000$	$N = 5,000$	$N = 10,000$
Two Equations h=200	0.055	0.054	0.053
Two Equations h=350	0.050	0.052	0.057
Two Equations h=500	0.045	0.050	0.052
One Equation h=200	0.053	0.054	0.051
One Equation h=350	0.051	0.051	0.048
One Equation h=500	0.049	0.049	0.054

Note: Results from Monte Carlo simulation using 5,000 replications and 1,000, 2,000, and 5,000 observations in each replication. We show the proportion of rejections of the null that each θ_1 is equal to the true value used in the simulations, using a significance level of 5%.

Table 8: Monte Carlo results for the proportion of rejections of $H_0 : \theta_2 = 0.15$

	$N = 2,000$	$N = 5,000$	$N = 10,000$
Two Equations h=200	0.063	0.056	0.051
Two Equations h=350	0.060	0.054	0.056
Two Equations h=500	0.054	0.054	0.052
One Equation h=200	0.058	0.057	0.050
One Equation h=350	0.059	0.043	0.054
One Equation h=500	0.055	0.050	0.050

Note: Results from Monte Carlo simulation using 5,000 replications and 1,000, 2,000, and 5,000 observations in each replication. We show the proportion of rejections of the null that each θ_2 is equal to the true value used in the simulations, using a significance level of 5%.

Because precise estimates for the parameters of interest are obtained in both cases, we conclude that using two separate equations, one for each kink, is a valid strategy.

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